A Mathematica Package for the Kinematic Analysis of Four-Bar Linkages

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1 Description of the Four-Bar Linkage

Link 1 or *AD* in Figure 1 is the fixed link of the four-bar mechanism. *Link 2* or *AB* is the driving link, *link 3* or *BC* the coupler link, and *link 4* or *CD* the output link. The location of the coupler point *P* is defined with respect to point *B* of link 3 by specifying the length of *BP* and the value of the angle α. The positions of the links are defined by the angles θ_2, θ_3 and θ_4. The angle θ_2 is the input angle, which, together with the link lengths r_1, r_2, r_3, r_4, determines the remaining angles θ_3 and θ_4. There are two possible closures of the linkage. In the programs, these are determined by setting the sgn variable equal to -1 or $+1$.

2 Position Analysis and Coupler Curves

To find the angles θ_3 and θ_4 for a given input angle θ_2 and given link lengths r_1, r_2, r_3, r_4, use the function call

```
Angles[r1, r2, r3, r4][theta2]
```

with `theta2` given in degrees. To find the possible limits imposed on the input link angle θ_2 by the proportions of the link lengths use

```
InputRange[r1, r2, r3, r4]
```

The Block `Coupler` is used to draw the coupler curve traced by P as the input link angle θ_2 takes on all its possible values. The call to `Coupler` is as follows

```
Coupler[r1, r2, r3, r4, r5, alpha, t2, sgn, options]
```

where `r5` is the length BP and `alpha` is the angle α of Figure 1 given in degrees. The angle `t2`, also specified in degrees, is the input link angle θ_2 and its value is used in drawing the linkage picture. The integer argument `sgn` is either -1 or $+1$ and determines the closure of the linkage for which the coupler curve is being drawn. The `options` argument should be `PSFile -> "outfile"` to save the plot to the file *outfile.ps*. If the `options` argument is omitted or is `PSFile -> None`, output is shown on the screen only.

2.1 Examples

1. `InputRange[200, 100, 250, 300]` returns `Interval[{0, 360.}]` so the input crank of the linkage is capable of making a complete revolution.

2. `InputRange[22, 45, 18, 58]` gives `Interval[{62.6715, 297.328}]` and the input link is a rocker confined to oscillations $62.67\,° \leq \theta_2 \leq 297.32\,°$.

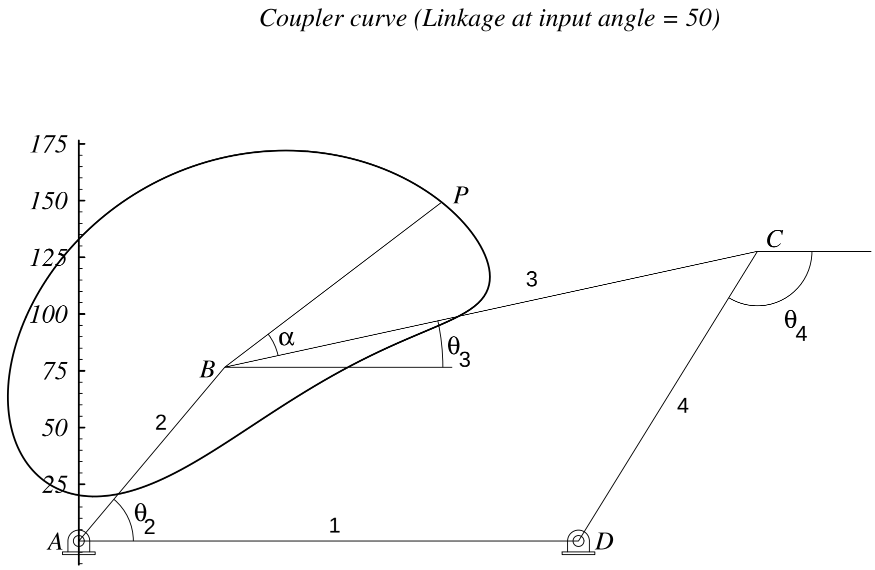


Figure 1: Position Variables for a Four-Bar Linkage

3. `InputRange[38, 54, 20, 67]` gives `Interval[{58.3908, 141.437}, {218.563, 301.609}]` so that two disjoint intervals $58.39\,^\circ \leq \theta_2 \leq 141.43\,^\circ$ and $218.56\,^\circ \leq \theta_2 \leq 301.61\,^\circ$ exist.

4. `Angles[200, 100, 250, 300][50]` gives

   ```
   {{50., 62.9206, -85.8108}, {50., -121.803, 26.9279}}
   ```

   Here the first entry of the list is for $sgn = -1$ and the second is for $sgn = +1$. Thus, if $sgn = +1$ and the input angle is $\theta_2 = 50\,^\circ$, then $\theta_3 = -121.80\,^\circ$ and $\theta_4 = 26.93\,^\circ$.

5. The result of

   ```
   Coupler[200, 100, 250, 300, 150, -45, 0., -1, PSFile -> "figure2"]
   ```

   is shown in Figure 2.

3 Velocity and Acceleration

`FourBar[r1, r2, r3, r4, sgn, options]` plots the angular velocities and accelerations of the links 3, 4 when the input link is driven at a constant angular velocity. The velocity and acceleration of a coupler point may also be plotted by setting the `PointMotion` option as `PointMotion -> {r, alpha}` where `r` is the length BP and `alpha` is the angle α in Figure 1. The other option in `FourBar` is `PSFile`. Here `PSFile` is a list with two elements. Its possible settings are

```
{None, None}     {"file1", None}    {None, "file2"}   {"file1", "file2"}
```

The specification `"file1"` as the first entry of `PSFile` causes angular velocity and angular acceleration plots to be saved in the PostScript files *file1.ps1* and *file1.ps2*. The angular velocity plots show ω_3/ω_2 and ω_4/ω_2 where ω_k denotes the angular velocity of link k. The angular acceleration plots are for ω_2 constant and show $\alpha_3/{\omega_2}^2$ and $\alpha_4/{\omega_2}^2$ where α_k is the angular acceleration of link k. If the first entry of `PSFile` is `None`, these plots are shown on the screen only.

The specification `"file2"` as the second entry of `PSFile` saves the PostScript file *file2.ps* containing the polar plots of velocity and acceleration of the point P whose location on link 3 has been specified by the `PointMotion`

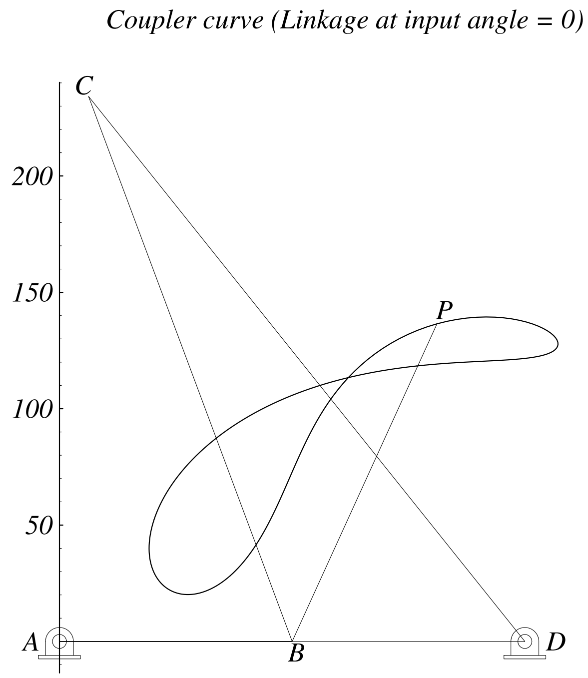


Figure 2: Example Coupler Curve Plot

option. If the `PointMotion` option is omitted, this part of the analysis is not done. If the `PointMotion` option is set and the second entry of `PSFile` is `None`, the velocity and acceleration curves for P are shown on the screen only. In these plots, a normalized velocity $\mathbf{v_P}/r_2\,\omega_2$ and a normalized acceleration $\mathbf{a_P}/r_2\,\omega_2{}^2$ are shown, where $\mathbf{v_P}$ and $\mathbf{a_P}$ are the velocity and acceleration of P.

3.1 Example

`FourBar[200, 100, 250, 300, -1, PointMotion -> {20, 30}, PSFile -> {"fg3", "fg4"}]` produces Figures 3, 4, and 5.

4 Notes

The internal function `data` stores the results of its calculations. This saves execution time if the same linkage is to be analyzed several times for diferent options or parameters. On the other hand, if data is stored for several different linkage dimensions, the increase in memory use may not be desirable, in which case the user may want to disable the data storing done by `data`.

Please send all bug reports or suggestions for improvement to the author's e-mail address.

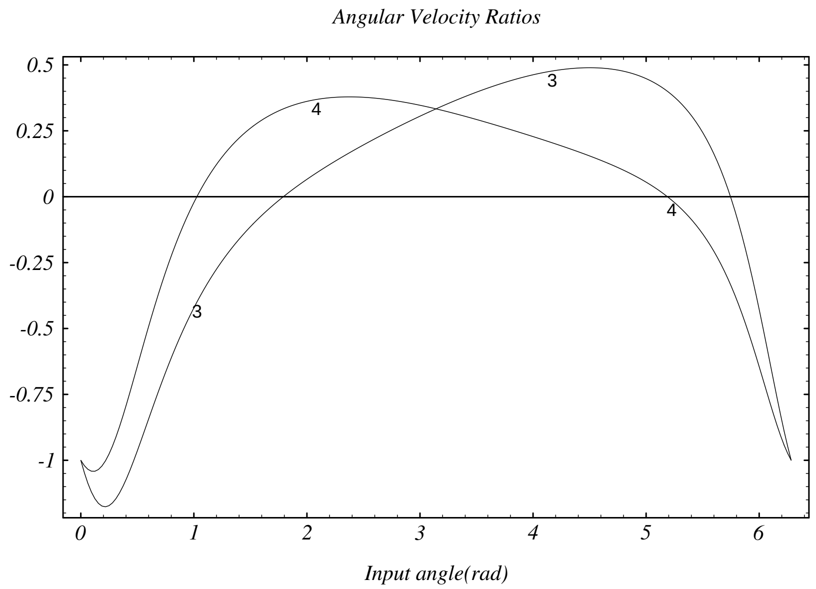


Figure 3: Angular Velocity Plots for a Four-Bar Linkage

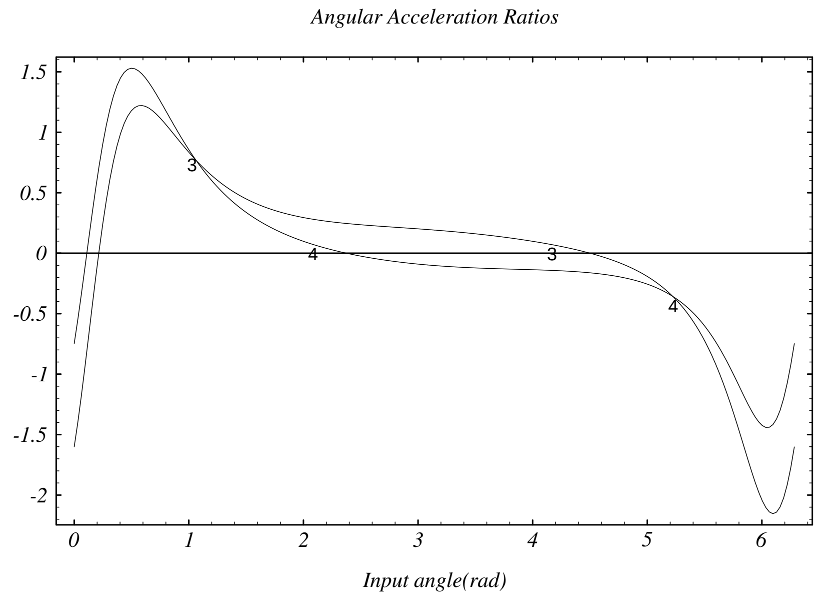
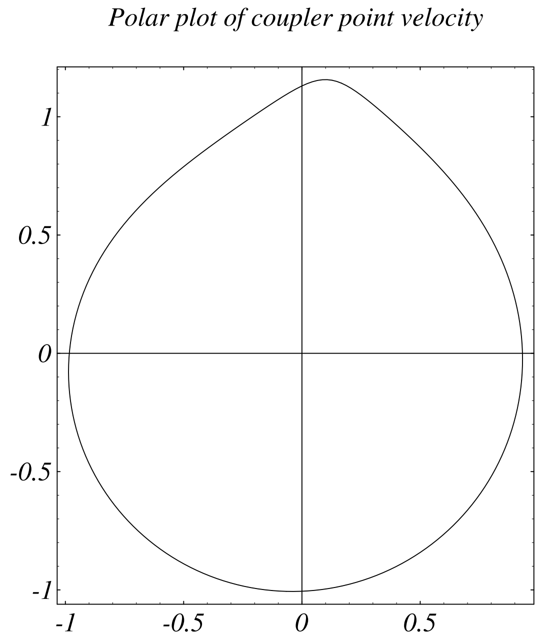


Figure 4: Angular Acceleration Plots for a Four-Bar Linkage



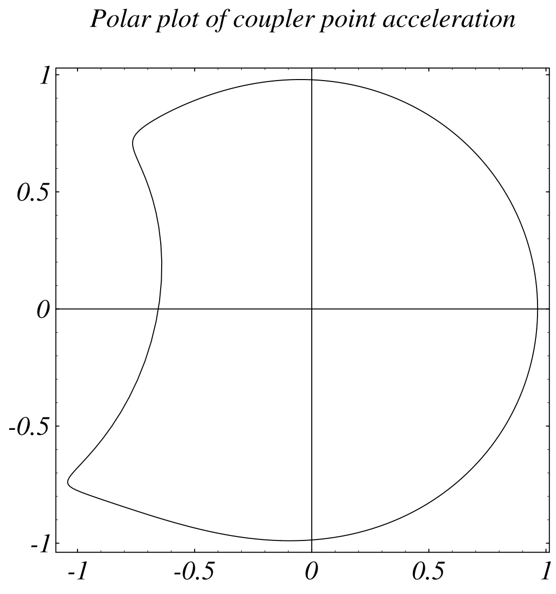


Figure 5: Velocity and Acceleration Plots for the Coupler Point