DYNAMICAL SYSTEMS WITH APPLICATIONS USING MATHEMATICA® SECOND EDITION

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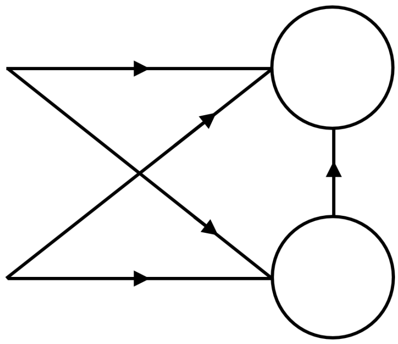

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Preface

Since the first printing of this book in 2007, Mathematica® has evolved from Mathematica version 6.0 to Mathematica version 11.1 in 2017. Accordingly, the second edition has been thoroughly updated and new material has been added. In this edition, there are many more applications, examples and exercises, all with solutions, and new sections on series solutions of ordinary differential equations and Newton fractals, have been added. There are also new chapters on delay differential equations, image processing, binary oscillator computing, and simulation with Wolfram SystemModeler.

This book provides an introduction to the theory of dynamical systems with the aid of Mathematica. It is written for both senior undergraduates and graduate students. Chapter 1 provides a tutorial introduction to Mathematica–new users should go through this chapter carefully whilst those moderately familiar and experienced users will find this chapter a useful source of reference. The first part of the book deals with continuous systems using differential equations, including both ordinary and delay differential equations (Chapters 2–12), the second part is devoted to the study of discrete systems (Chapters 13–17), and Chapters 18–22 deal with both continuous and discrete systems. Chapter 23 gives examples of coursework and also lists three Mathematica-based examinations to be sat in a computer laboratory with access to Mathematica. Chapter 24 lists answers to all of the exercises given in the book. It should be pointed out that dynamical systems theory is not limited to these topics but also encompasses partial differential equations, integral and integro-differential equations, and stochastic systems, for instance. References [1]–[6] given at the end of the Preface provide more information for the interested reader. The author has gone for breadth of coverage rather than fine detail and theorems with proofs are kept at a minimum. The material is not clouded by functional analytic and group theoretical definitions, and so is intelligible to readers with a general mathematical background. Some of the topics covered are scarcely covered elsewhere. Most of the material in Chapters 9-12 and 16-22 is at postgraduate level and has been influenced by the author's own research interests. There is more theory in these chapters than in the rest of the book since it is not easily accessed anywhere else. It has been found that these chapters are especially useful as reference material for senior undergraduate project work. The theory in other chapters of the book is dealt with more comprehensively in other texts, some of which may be found in the references section of the corresponding chapter. The book has a very hands-on approach and takes the reader from the basic theory right through to recently published research material.

Mathematica is extremely popular with a wide range of researchers from all sorts of disciplines, it has a very user-friendly interface and has extensive visualization and numerical computation capabilities. It is an ideal package to adopt for the study of nonlinear dynamical systems; the numerical algorithms work very quickly, and complex pictures can be plotted within seconds. The Wolfram SystemModeler™ package is used for simulating dynamical processes. It is as close as one can get to building apparatus and investigating the output for a given input without the need for an actual physical model. For this reason, SystemModeler is very popular in the field of engineering.

The first chapter provides an efficient tutorial introduction to Mathematica. New users will find the tutorials will enable them to become familiar with Mathematica within a few hours. Both engineering and mathematics students appreciate this method of teaching and I have found that it generally works well with one staff member to about twenty students in a computer laboratory. In most cases, I have chosen to list the Mathematica notebook commands at the end of each chapter, this avoids unnecessary cluttering in the text. The Mathematica programs have been kept as simple as possible and should run under later versions of the package. All files for the book (including updates) can even be downloaded from the Web at

http://library.wolfram.com/infocenter/Books/6881/.

Readers will find that they can reproduce the figures given in the text, and then it is not too difficult to change parameters or equations to investigate other systems.

Chapters 2–12 deal with continuous dynamical systems. Chapters 2 and 3 cover some theory of ordinary differential equations and applications to models in the real world are given. The theory of differential equations applied to chemical kinetics and electric circuits is introduced in some detail. The memristor is introduced and one of the most remarkable stories in the history of mathematics is relayed. Chapter 2 ends with the existence and uniqueness theorem for the solutions of certain types of differential equations. The theory behind the construction of phase plane portraits for two-dimensional systems is dealt with in Chapter 3. Applications are taken from chemical kinetics, economics, electronics, epidemiology, mechanics, and population dynamics. The modeling of the populations of interacting species are discussed in some detail in Chapter 4 and domains of stability are discussed for the first time. Limit cycles, or isolated periodic solutions, are introduced in Chapter 5. Since we live in a periodic world, these are the most common type of solution found when modeling nonlinear dynamical systems. They appear extensively when modeling both the technological and natural sciences. Hamiltonian, or conservative, systems and stability are discussed in Chapter 6, and Chapter 7 is concerned with how planar systems vary depending upon a parameter. Bifurcation, bistability, multistability, and normal forms are discussed.

The reader is first introduced to the concept of chaos in continuous systems in Chapters 8 and 9, where three-dimensional systems and Poincaré maps are investigated. These higher-dimensional systems can exhibit strange attractors and chaotic dynamics. One can rotate the three-dimensional objects in Mathematica and plot time series plots to get a better understanding of the dynamics involved. Once again, the theory can be applied to chemical kinetics (including stiff systems), electric circuits, and epidemiology; a simplified model for the weather is also briefly discussed. Chapter 9 deals with Poincaré first return maps that can be used to untangle complicated interlacing trajectories in higher-dimensional spaces. A periodically driven nonlinear pendulum is also investigated by means of a nonautonomous differential equation. Both local and global bifurcations are investigated in Chapter 10. The main results and statement of the famous second part of David Hilbert's sixteenth problem are listed in Chapter 11. In order to understand these results, Poincaré compactification is introduced. There is some detail on Liénard systems, in particular, in this part of the book, but they do have a ubiquity for systems in the plane. Chapter 12 provides an introduction to delay differential equations with applications in biology, nonlinear optics and other dynamical systems.

Chapters 13–17 deal with discrete dynamical systems. Chapter 13 starts with a general introduction to iteration and linear recurrence (or difference) equations. The bulk of the chapter is concerned with the Leslie model used to investigate the population of a single species split into different age classes. Harvesting and culling policies are then investigated and optimal solutions are sought. Nonlinear discrete dynamical systems are dealt with in Chapter 14. Bifurcation diagrams, chaos, intermittency, Lyapunov exponents, periodicity, quasiperiodicity, and universality are some of the topics introduced. The theory is then applied to real-world problems from a broad range of disciplines including population dynamics, biology, economics, nonlinear optics, and neural networks. Chapter 15 is concerned with complex iterative maps, Julia sets and the now-famous Mandelbrot set are plotted. Basins of attraction are investigated for these complex systems and Newton fractals are introduced. As a simple introduction to optics, electromagnetic waves and Maxwell's equations are studied at the beginning of Chapter 16. Complex iterative equations are used to model the propagation of light waves through nonlinear optical fibers. A brief history of nonlinear bistable optical resonators is discussed and the simple fiber ring resonator is analyzed in particular. Chapter 16 is devoted to the study of these optical resonators and phenomena such as bistability, chaotic attractors, feedback, hysteresis, instability, linear stability analysis, multistability, nonlinearity, and steady-states are discussed. The first and second iterative methods are defined in this chapter. Some simple fractals may be constructed using pencil and paper in Chapter 17, and the concept of fractal dimension is introduced. Fractals may be thought of as identical motifs repeated on ever-reduced scales. Unfortunately, most of the fractals appearing in nature are not homogeneous but are more heterogeneous, hence the need for the multifractal theory given later in the chapter. It has been found that the distribution of stars and galaxies in our universe are multifractal, and there is even evidence of multifractals in rainfall, stock markets, and heartbeat rhythms. Applications in geoscience, materials science, microbiology, and image processing are briefly discussed. Chapter 18 provides a brief introduction to image processing which is being used more and more by a diverse range of scientific disciplines, especially medical imaging. The fast Fourier transform is introduced and has a wide range of applications throughout the realms of science.

Chapter 19 is devoted to the new and exciting theory behind chaos control and synchronization. For most systems, the maxim used by engineers in the past has been "stability good, chaos bad", but more and more nowadays this is being replaced with "stability good, chaos better". There are exciting and novel applications in cardiology, communications, engineering, laser technology, and space research, for example. A brief introduction to the enticing field of neural networks is presented in Chapter 20. Imagine trying to make a computer mimic the human brain. One could ask the question: In the future will it be possible for computers to think and even be conscious? The human brain will always be more powerful than traditional, sequential, logic-based digital computers and scientists are trying to incorporate some features of the brain into modern computing. Neural networks perform through learning and no underlying equations are required. Mathematicians and computer scientists are attempting to mimic the way neurons work together via synapses; indeed, a neural network can be thought of as a crude multidimensional model of the human brain. The expectations are high for future applications in a broad range of disciplines. Neural networks are already being used in machine learning and pattern recognition (computer vision, credit card fraud, prediction and forecasting, disease recognition, facial and speech recognition), the consumer home entertainment market, psychological profiling, predicting wave over-topping events, and control problems, for example. They also provide a parallel architecture allowing for very fast computational and response times. In recent years, the disciplines of neural networks and nonlinear dynamics have increasingly coalesced and a new branch of science called neurodynamics is emerging. Lyapunov functions can be used to determine the stability of certain types of neural network. There is also evidence of chaos, feedback, nonlinearity, periodicity, and chaos synchronization in the brain.

Chapter 21 focuses on binary oscillator computing, the subject of UK, International and Taiwanese patents. The author and his co-inventor, Jon Borresen, came up with the idea when modeling connected biological neurons. Binary oscillator technology can be applied to the design of Arithmetic Logic Units (ALU)s, memory and other basic computing components. It has the potential to provide revolutionary computational speed-up, energy saving and novel applications and may be applicable to a variety of technological paradigms including biological neurons, Complementary Metal-Oxide-Semiconductor (CMOS), memristors, optical oscillators, and superconducting materials. The research has the potential for MMU and industrial partners to develop super fast, low-power computers, and may provide an assay for neuronal degradation for brain malfunctions such as Alzheimer's, epilepsy and Parkinson's disease!

Examples of SystemModeler, referred to in earlier chapters of the book, are presented in Chapter 22. It is possible to change the type of input into the system, or parameter values, and investigate the output very quickly. This is as close as one can get to experimentation without the need for expensive equipment.

Examples of coursework and three examination-type papers are listed in Chapter 23, and a set of outline solutions for the book is listed in Chapter 24.

Both textbooks and research papers are presented in the list of references. The textbooks can be used to gain more background material, and the research papers have been given to encourage further reading and independent study.

This book is informed by the research interests of the author which are currently nonlinear ordinary differential equations, nonlinear optics, multifractals, neural networks, and binary oscillator computing. Some references include recently published research articles by the author along with two patents.

The prerequisites for studying dynamical systems using this book are undergraduate courses in linear algebra, real and complex analysis, calculus, and ordinary differential equations; a knowledge of a computer language such as C or Fortran would be beneficial but not essential.

Recommended Textbooks

[1] K. Adzievski and A.H. Siddiqi, *Introduction to Partial Differential Equations for Scientists and Engineers using Mathematica, Second Edition*, Chapman & Hall/CRC (Applied Mathematics & Nonlinear Science), New York, 2016.

[2] B. Bhattacharya and M. Majumdar, *Random Dynamical Systems in Finance*, Chapman & Hall/CRC, New York, 2016.

[3] L.C. de Barros, R.C. Bassanezi and W.A. Lodwick, *A First Course in Fuzzy Logic, Fuzzy Dynamical Systems, and Biomathematics: Theory and Applications*, Springer, New York, 2016.

[4] V. Volterra, *Theory of Functionals and of Integral and Integro-Differential Equations*, Dover Publications, New York, 2005.

[5] J. Mallet-Paret (Editor), J. Wu (Editor), H. Zhu (Editor), Y. Yi (Editor), *Infinite Dimensional Dynamical Systems (Fields Institute Communications)*, Springer, New York, 2013.

[6] C. Bernido, M.V. Carpio-Bernido, M. Grothaus et al., *Stochastic and Infinite Dimensional Analysis*, Birkhäuser, New York, 2016.

I would like to express my sincere thanks to Wolfram for supplying me with the latest versions of Mathematica and SystemModeler. Thanks also go to the reviewer of this book for his thorough checking and all of the reviewers from the other editions of my books. Special thanks go to Birkhäuser and Springer International Publishers. Finally, thanks to my family and especially my wife Gaynor, and our children, Sebastian and Thalia, for their continuing love, inspiration, and support.

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