TU/e BioMIM

Medical Image Processing with Orientation Scores

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Outline

- **About our Research Group**
- Orientation Scores
- Diffusion in Orientation Scores
- Stochastic Completion Fields
- Using Mathematica

The Biomedical Image Analysis group

Mathematica in the BioMIM lab

Starring:
Petr Šereda (Pilsen, Czech Republic)
Tim Peeters (Echt, The Netherlands)

Our Mathematica Infrastructure

- Full campus license for Mathematica
- The need for “bigmath” kernel servers
  - Bigmath1: Tyan TX46, 4x Opteron 2.2Ghz, 32GB
  - Bigmath2: Tyan VX50, 4x Dualcore Opteron 2.2Ghz, 64GB
  - + 3 older servers
- Use of ParallelMathematica

TU/e BioMedical Image Analysis MATHEMATICA® COMPUTING CLUSTER

- Home
- Instructions
- Links

- Math1
- Math2
- Math3
- Bigmath1
- Bigmath2

bigmath2

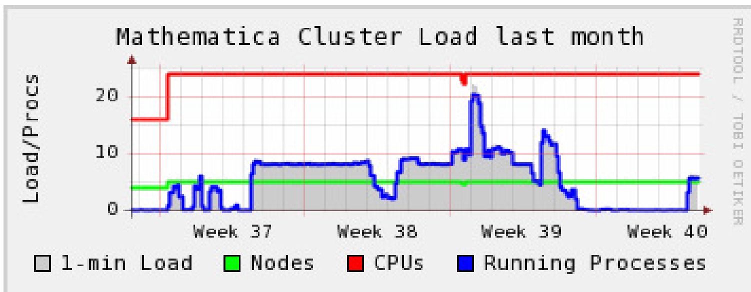
Bigmath2 is a Tyan VX50 with 4 Dualcore Opteron 2.2Ghz cpus and 64GB of ram. Its hostname is bigmath2.bmt.tue.nl

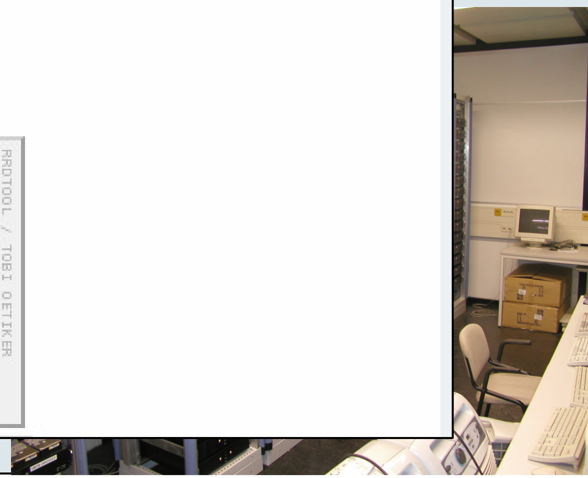
Mathematica kernels currently running on bigmath2.bmt.tue.nl: 15

Username(cpu usage)
fkanters(0.6%)
fkanters(1.2%)
fkanters(0.0%)
fkanters(0.6%)
fkanters(17.5%)
fkanters(17.9%)
fkanters(0.0%)
bjanssen(69.8%)
bjanssen(69.7%)
bjanssen(82.2%)
bjanssen(69.8%)
bjanssen(70.0%)
bjanssen(70.3%)
bjanssen(70.5%)
bjanssen(70.7%)

Performance statistics

Show statistics from: Last month





MathVisionTools

Computer Vision Library for Mathematica:

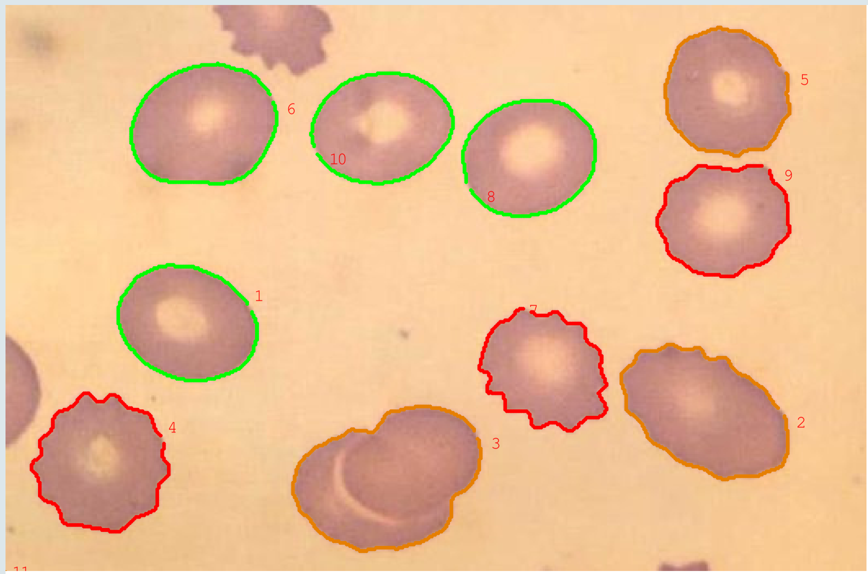
- Gaussian derivatives
- Geometry driven diffusion
- Orientation score functions
- Image transformations
- DICOM import/export

www.mathvisiontools.net

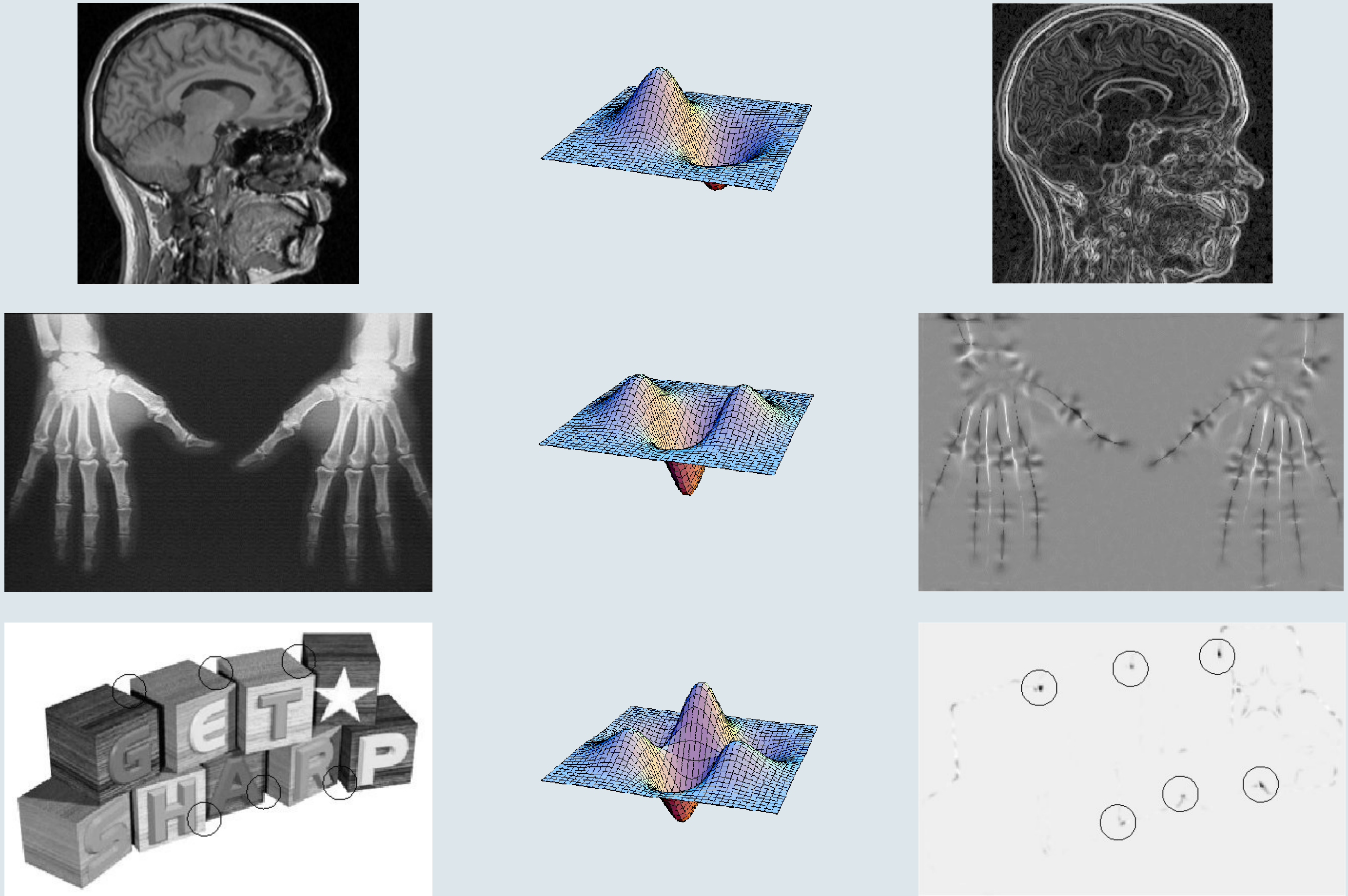
Mathematica in Bachelor Education

Image Analysis for Pathology.

- Groups of 8 2nd year students
- “Invent” image analysis algorithms in *Mathematica*
- Competitive element
- 6 weeks project



Example: Differential invariants



1st order
(edges)

2nd order
(ridges)

3rd order
(T-junctions)

For example
Rotation invariant
T-junction detection:

$$\frac{1}{(L_x^2+L_y^2)^3}\left(-L_{xxy}L_y^5+L_y^4\left(2L_{xy}^2-L_x\left(L_{xxx}-2L_{xyy}\right)+L_{xx}L_{yy}\right)+\right.$$
$$L_x^4\left(2L_{xy}^2-L_xL_{xyy}+L_{xx}L_{yy}\right)+L_x^2L_y^2\left(3L_{xx}^2-8L_{xy}^2+L_x\left(-L_{xxx}+L_{xyy}\right)-4L_{xx}L_{yy}+3L_{yy}^2\right)+$$
$$\left.L_xL_y^3\left(6L_{xy}\left(L_{xx}-L_{yy}\right)+L_x\left(L_{xxy}-L_{yyy}\right)\right)+L_x^3L_y\left(6L_{xy}\left(-L_{xx}+L_{yy}\right)+L_x\left(2L_{xxy}-L_{yyy}\right)\right)\right)$$

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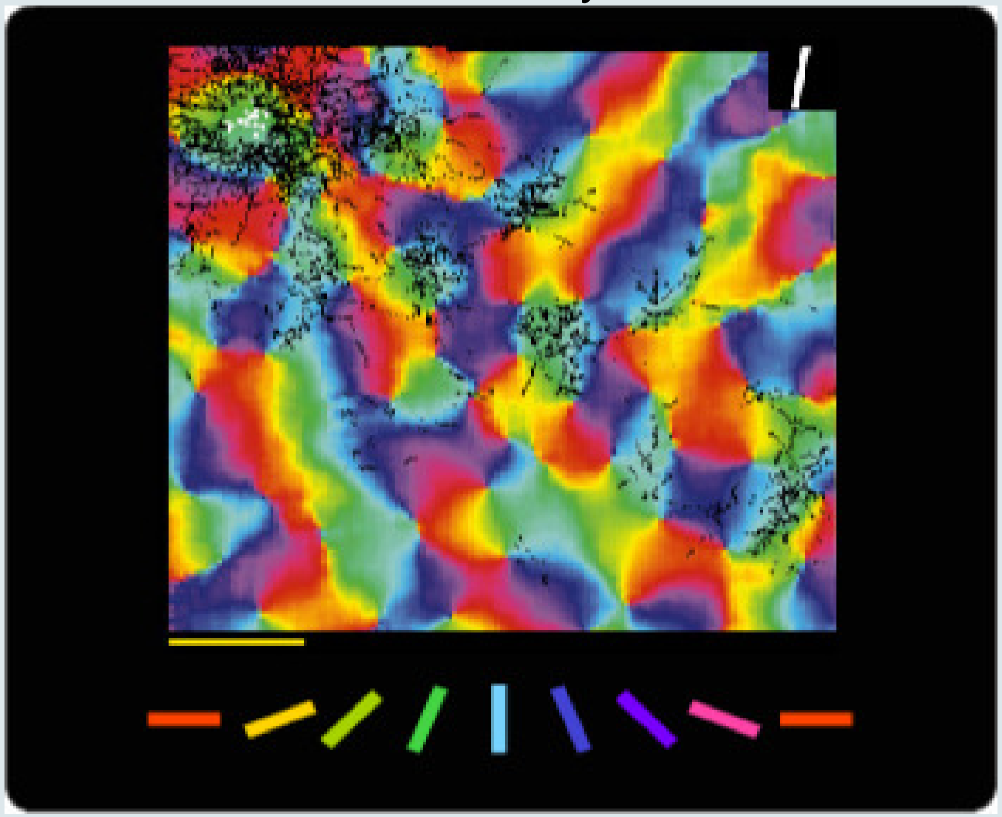
Biological Inspiration

1. The retina contains receptive fields of varying sizes → *multi-scale* sampling device
2. Primary visual cortex is *multi-orientation*

Measurement in Primary Visual Cortex

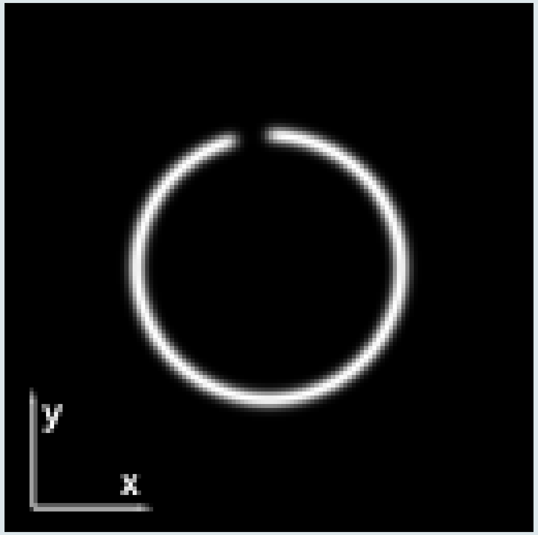


Bosking et al., J. Neuroscience 17:2112-2127, 1997

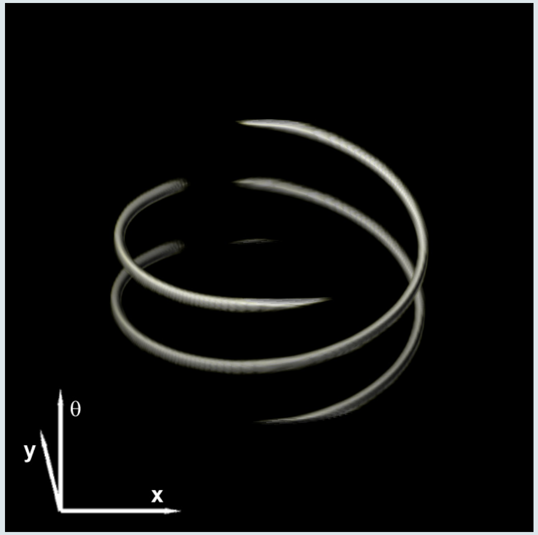
- Cells in the primary visual cortex are orientation-specific
- Strong connectivity between cells that respond to (nearly) the same orientation

Orientation Scores

From 2D image $f(x,y)$ to orientation score $U_f(x,y,\theta)$ with position (x,y) and orientation θ



$$f(x,y) \xrightarrow{W_\psi} U_f(x,y,\theta)$$

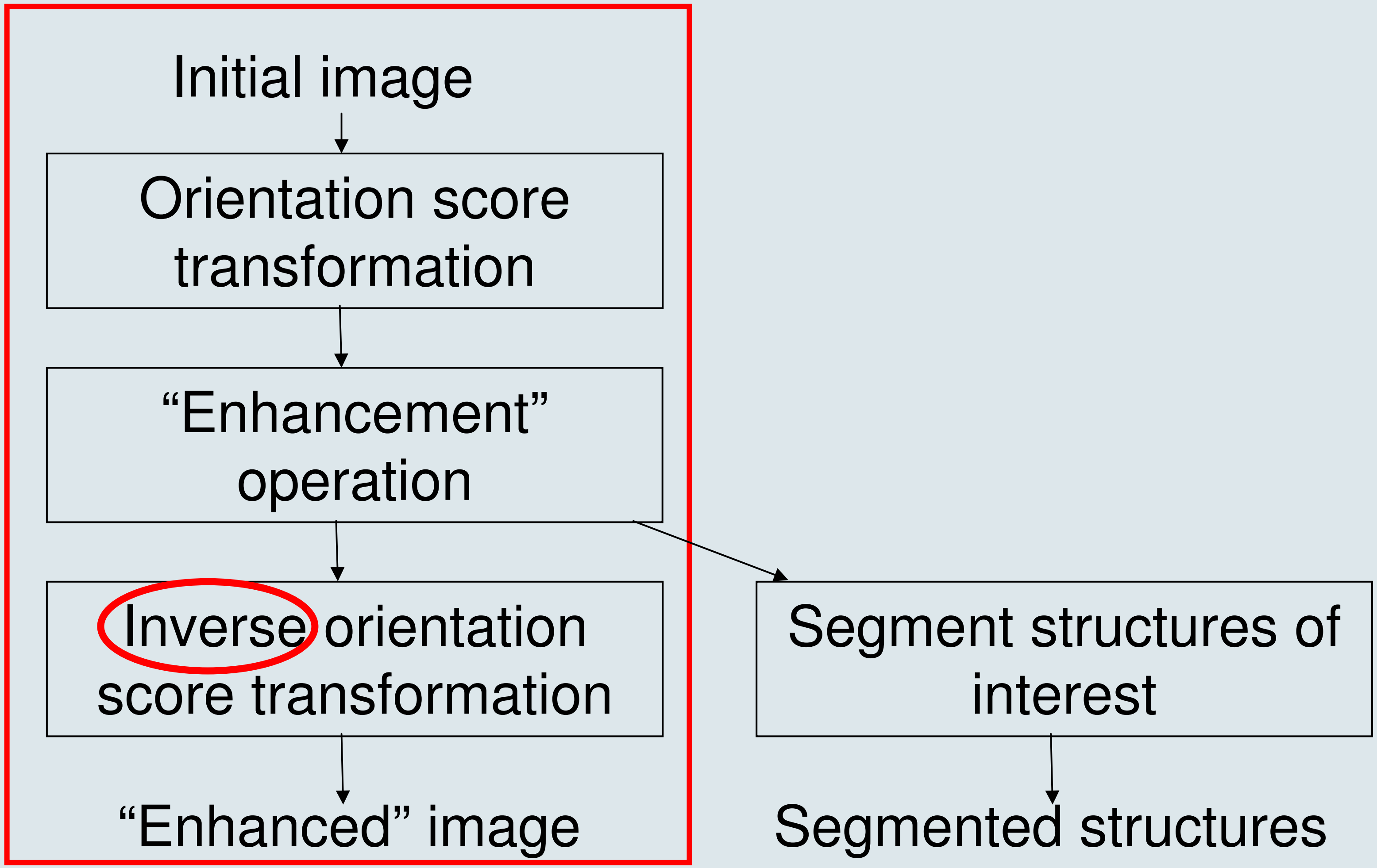


$$(W_\psi[f])(\mathbf{x},\theta) = U_f(\mathbf{x},\theta) = \int_{\mathbb{R}^2} \psi(R_\theta^{-1}(\mathbf{x}'-\mathbf{x}))f(\mathbf{x}')\, d\mathbf{x}'$$

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

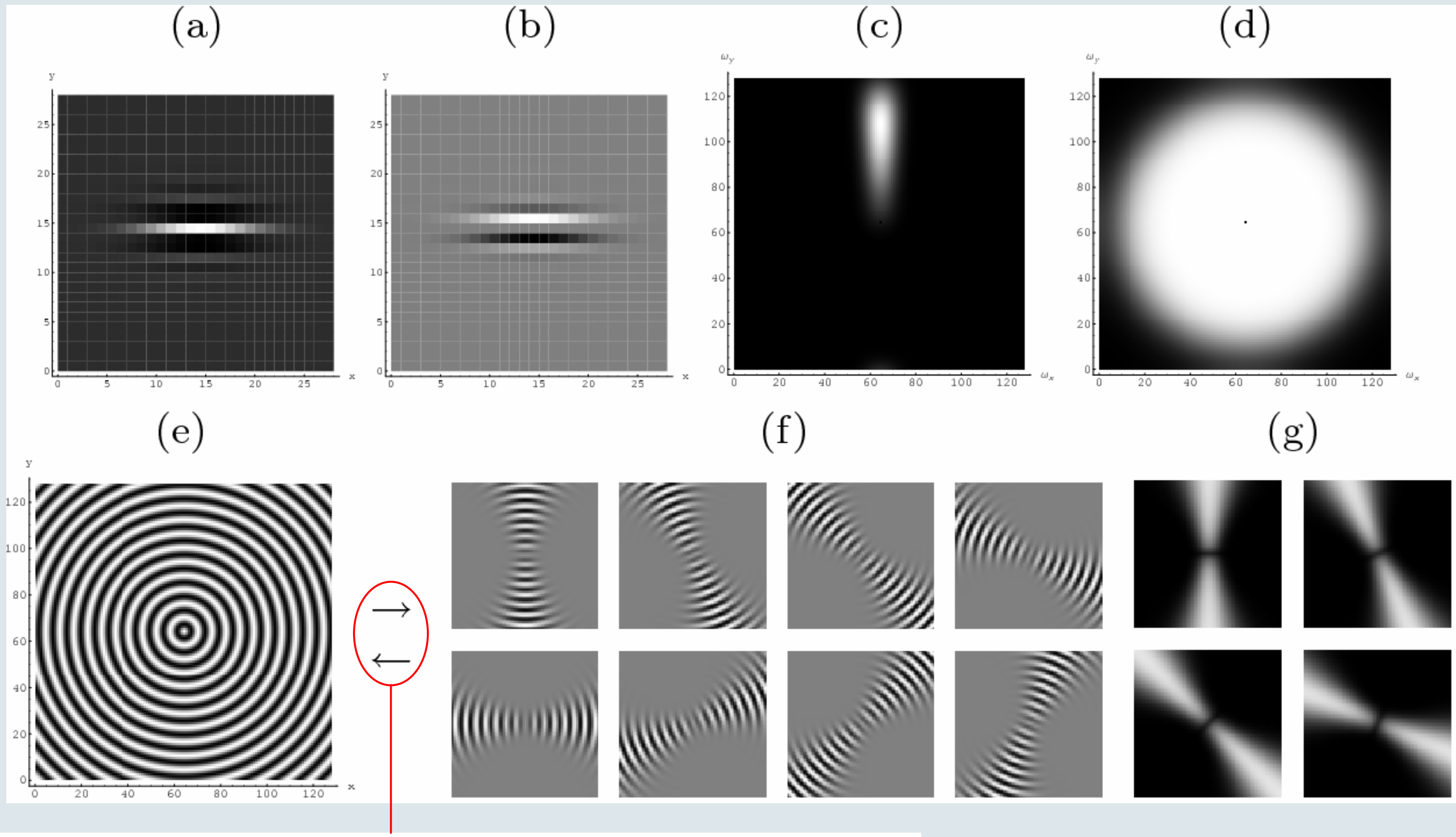
An orientation score is a function on the *Euclidean motion group*

Our approach: Image Processing *via* Orientation Scores



Invertible Orientation Score Transformation

Design considerations: reconstruction, directional, spatial localization, quadrature




```
os1 = CKOrientationScoreTransform[img1, k, sφ, q, t, ss];
img1back = Plus @@ os1;
```

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The Diffusion Equation on Images

$$\begin{cases} \frac{\partial}{\partial t} u = \nabla \cdot \mathbf{D}\, \nabla u \\ u(\mathbf{x}; 0) = f(\mathbf{x}) \end{cases}$$

f = image

u = scale space of image

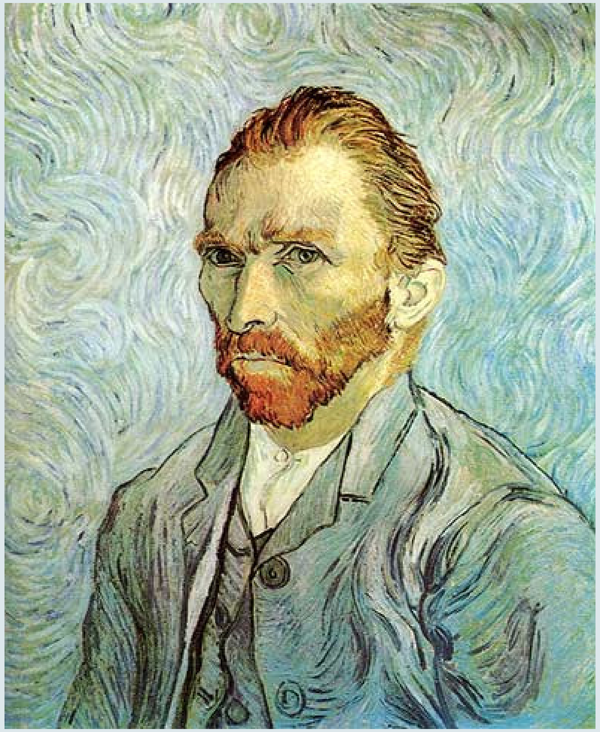
D = diffusion tensor

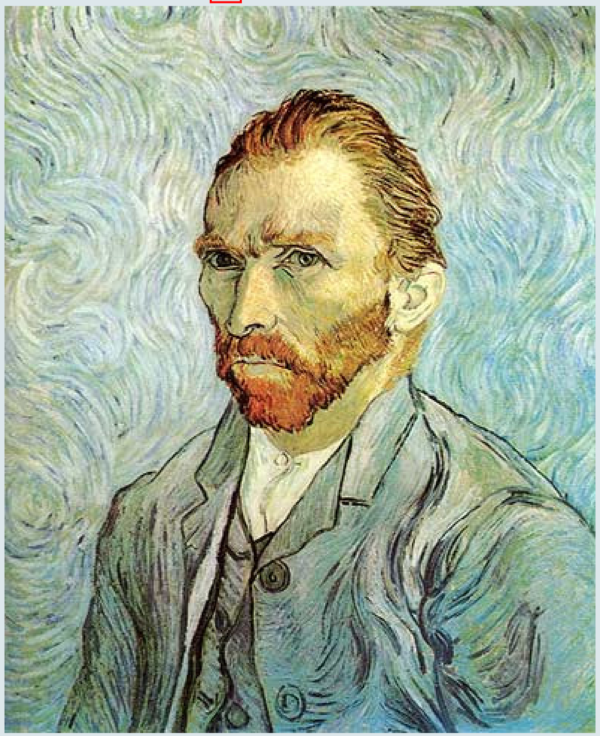
$$\nabla = \left(\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \right)^T$$

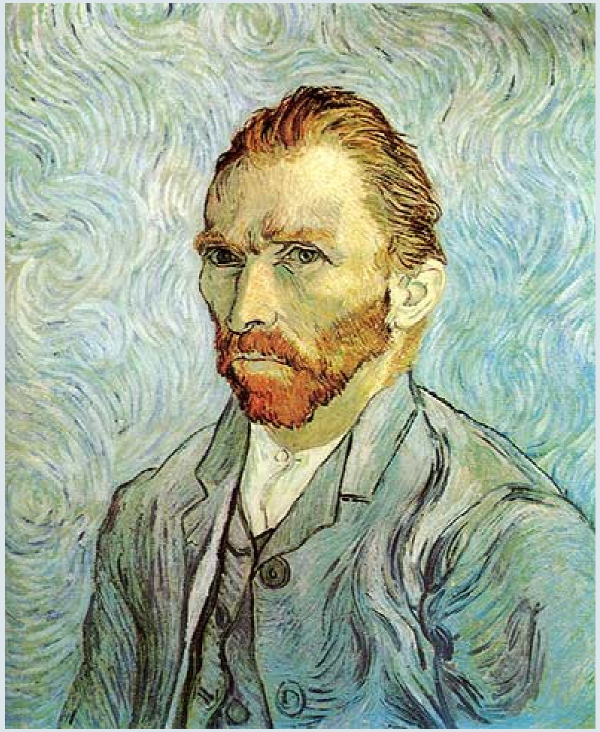
Linear diffusion
$\mathbf{D} = \mathbf{I}$

Perona&Malik
$\mathbf{D}(\mathbf{x}) = g(|\nabla u(\mathbf{x})|)\, \mathbf{I}$

Coherence-enhancing diff.
$\mathbf{D}(\mathbf{x}) = s(\mathbf{x})\, \mathbf{v}(\mathbf{x})\, \mathbf{v}^T(\mathbf{x}) + \alpha \mathbf{I}$







$t = 0$

The Diffusion Equation on Images

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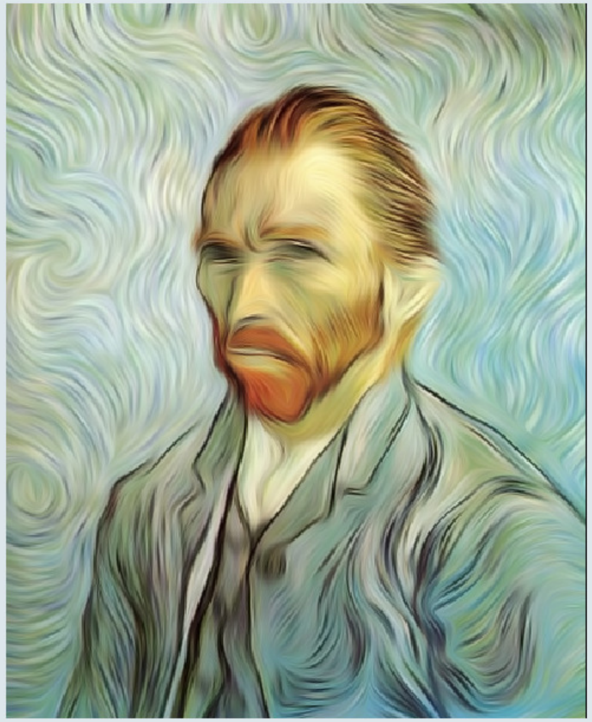


$t = 10$

Diffusion in orientation scores

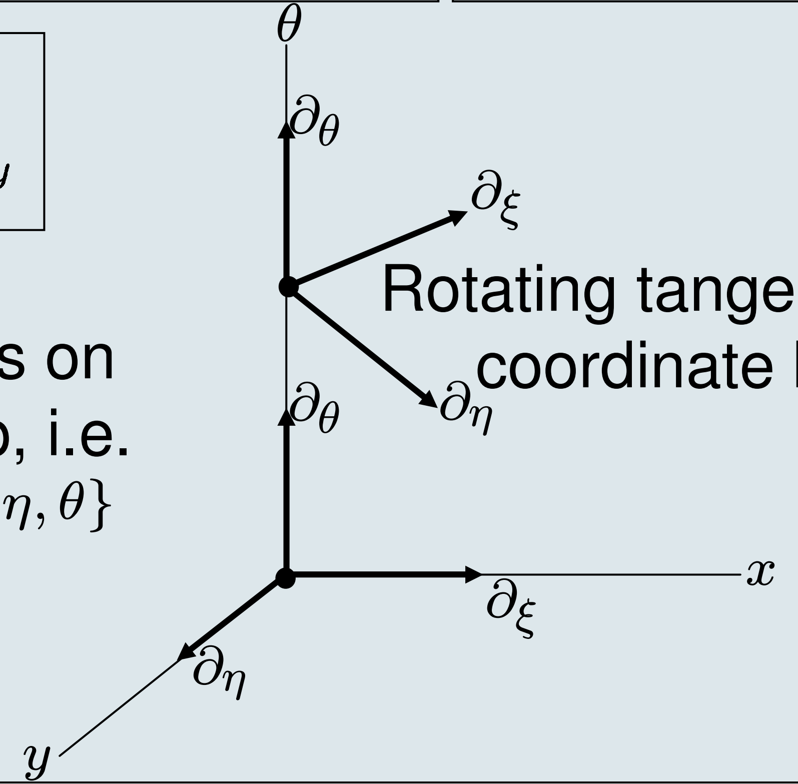
$$\partial_t u = \begin{pmatrix}\partial_\theta & \partial_\xi & \partial_\eta\end{pmatrix} \begin{pmatrix} D'_{11} + D_{22}\kappa^2 & D_{22}\kappa & 0 \\ D_{22}\kappa & D_{22} & 0 \\ 0 & 0 & D_{33} \end{pmatrix} \begin{pmatrix}\partial_\theta \\ \partial_\xi \\ \partial_\eta\end{pmatrix} u$$

- Left-invariant derivatives
- Diffusion in orientation
- Diffusion tangent to oriented structures
- Diffusion orthogonal to oriented structures
- curvature
- Evolving orientation score

$$\partial_\xi = \cos\theta\,\partial_x + \sin\theta\,\partial_y$$
$$\partial_\eta = -\sin\theta\,\partial_x + \cos\theta\,\partial_y$$

∂_ξ, ∂_η, and ∂_θ are left-invariant derivatives on Euclidean motion group, i.e.

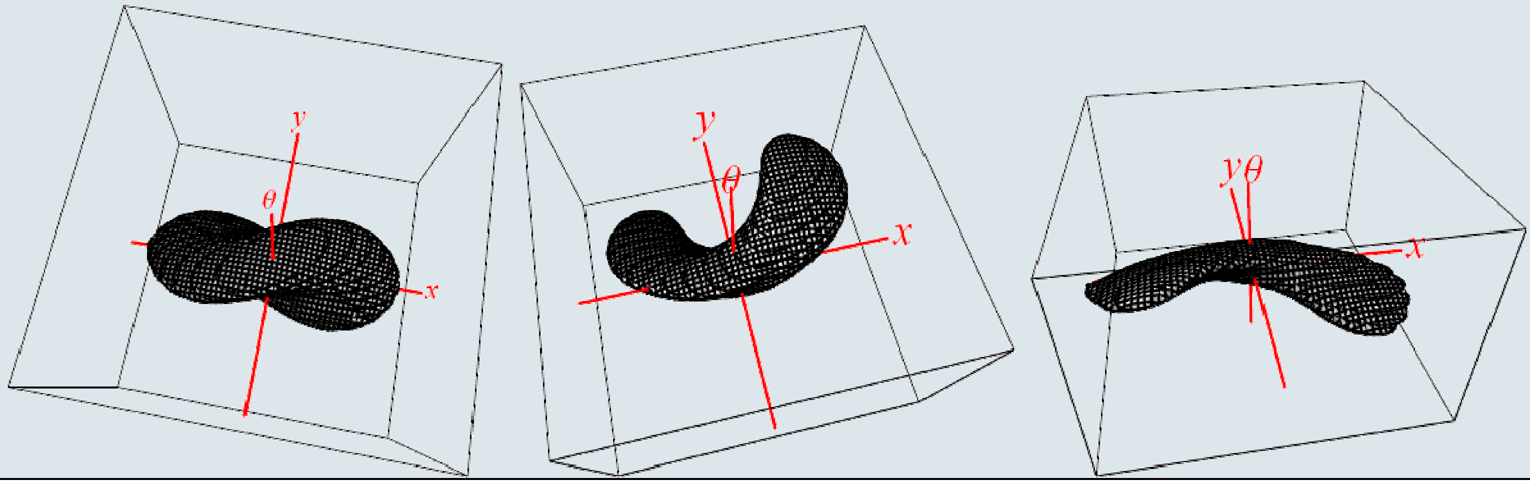
$$\mathcal{L}_g \partial_i U = \partial_i \mathcal{L}_g U, \;\; i \in \{\xi, \eta, \theta\}$$



Rotating tangent space coordinate basis

Example diffusion kernels

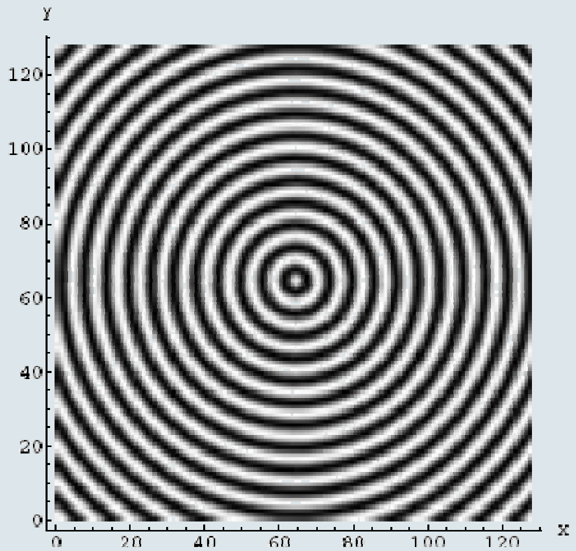
$$\partial_t u = \begin{pmatrix}\partial_\theta & \partial_\xi & \partial_\eta\end{pmatrix} \begin{pmatrix} D'_{11} + D_{22}\kappa^2 & D_{22}\kappa & 0 \\ D_{22}\kappa & D_{22} & 0 \\ 0 & 0 & D_{33} \end{pmatrix} \begin{pmatrix}\partial_\theta \\ \partial_\xi \\ \partial_\eta\end{pmatrix} u$$



How to Choose Conductivity Coefficients

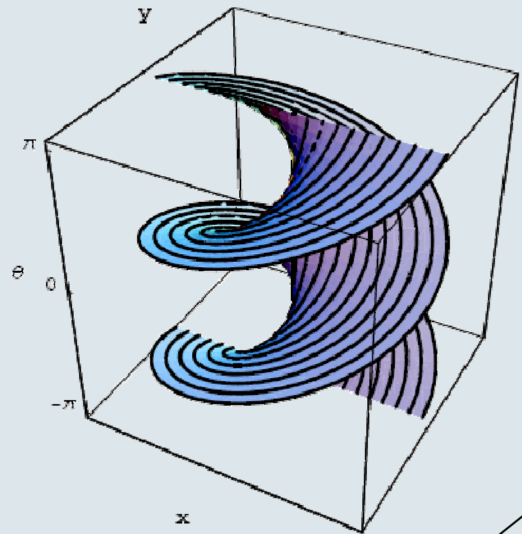
$$\partial_t u = \begin{pmatrix} \partial_\theta & \partial_\xi & \partial_\eta \end{pmatrix} \begin{pmatrix} D'_{11} + D_{22}\kappa^2 & D_{22}\kappa & 0 \\ D_{22}\kappa & D_{22} & 0 \\ 0 & 0 & D_{33} \end{pmatrix} \begin{pmatrix} \partial_\theta \\ \partial_\xi \\ \partial_\eta \end{pmatrix} u$$

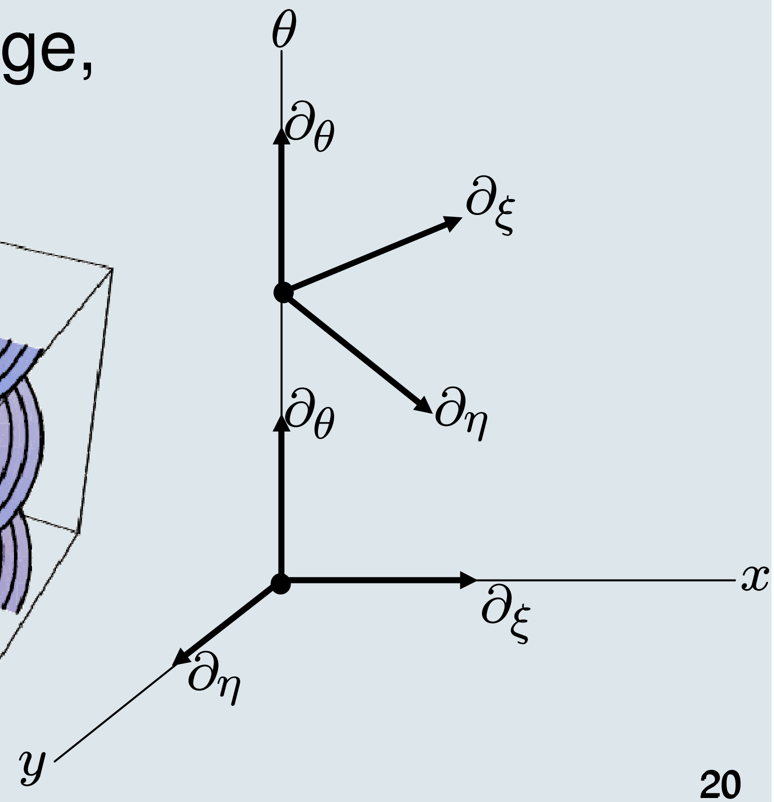
- Oriented regions: D'_{11} and D_{33} small, D_{22} large and κ according to estimate
- Non-oriented regions: D'_{11} large, $D_{22}=D_{33}$ large, $\kappa = 0$



$\longrightarrow$

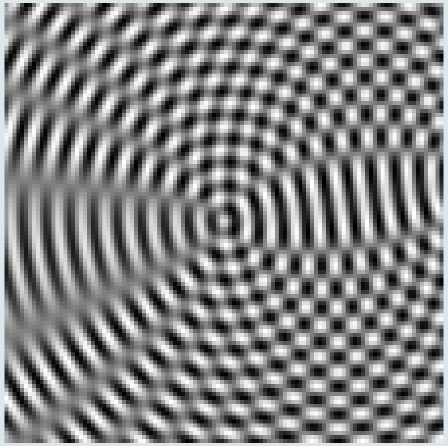
$\longleftarrow$



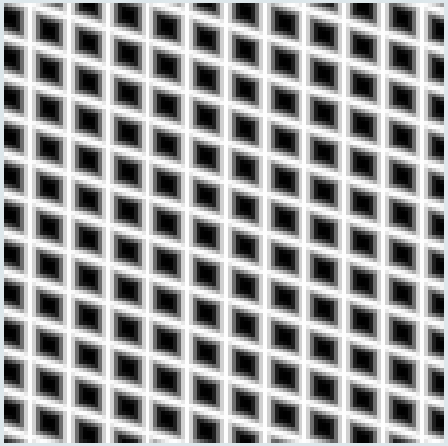


Results

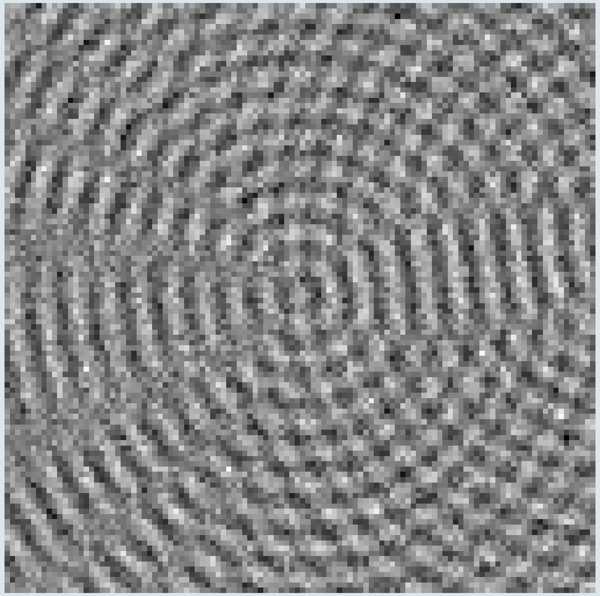
$t = 0$

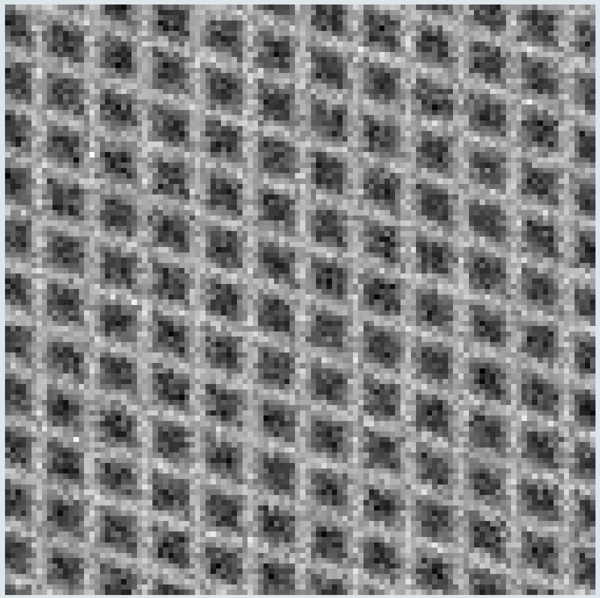


Size: 128 x 128 x 64

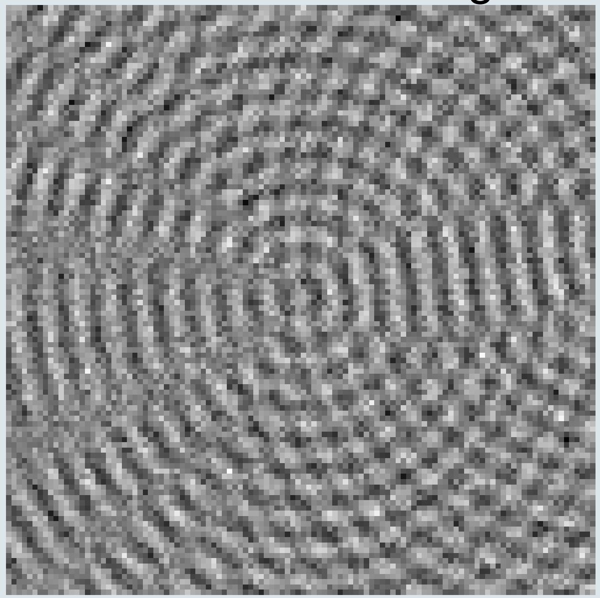


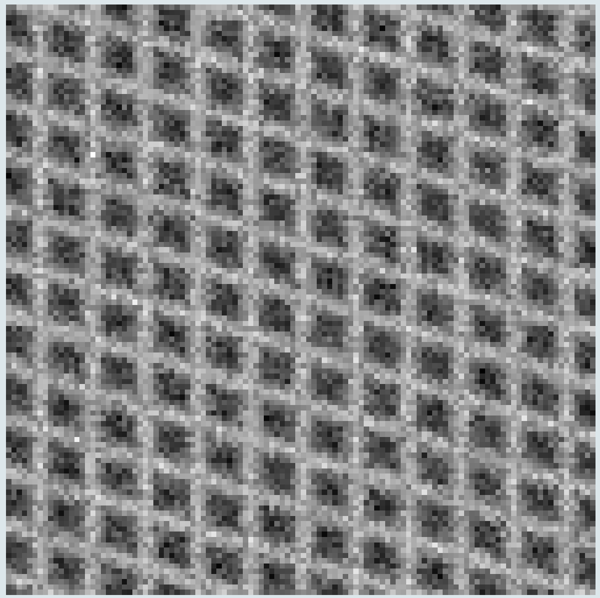
Diffusion in orientation score





Coherence enhancing diffusion



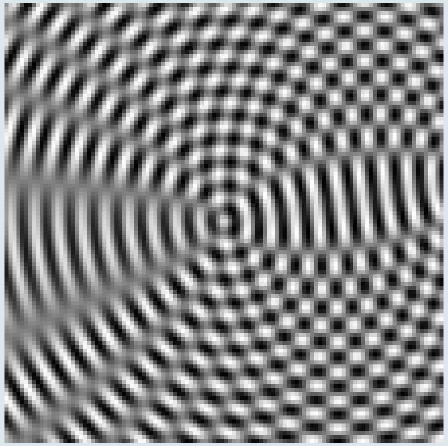
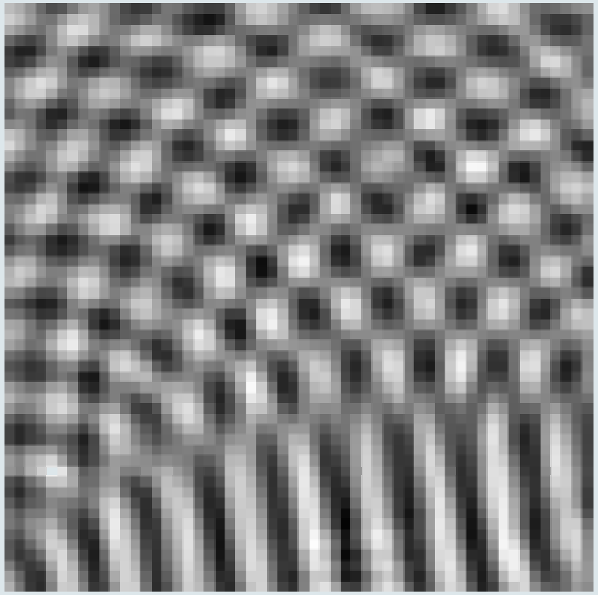


Results

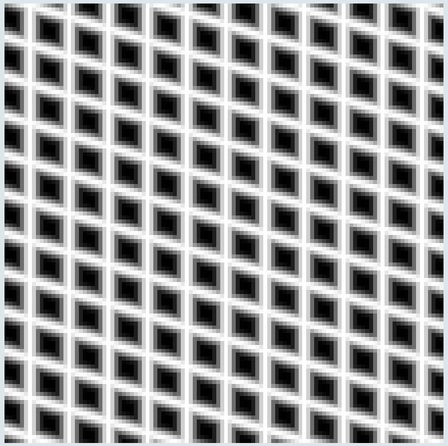
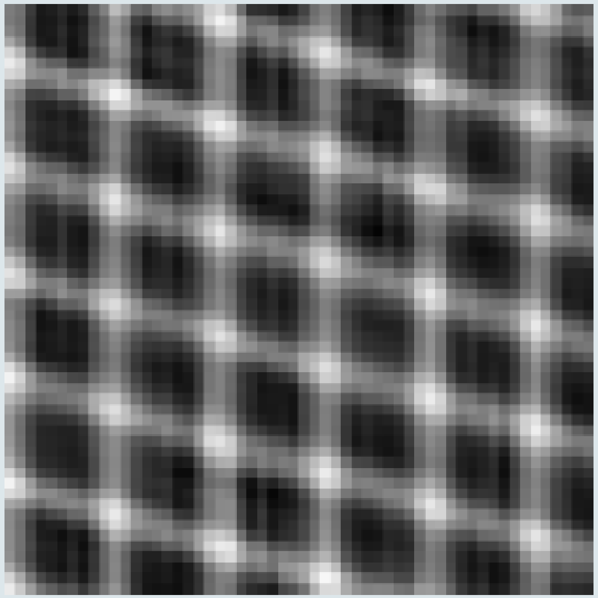
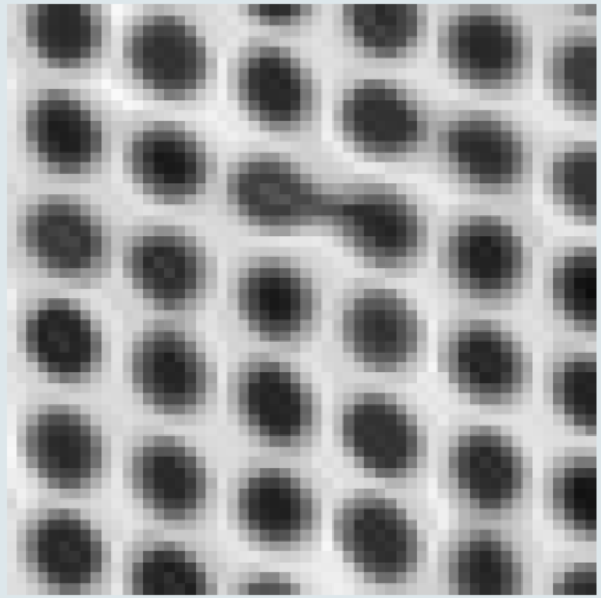
$t = 10$

Diffusion in orientation score

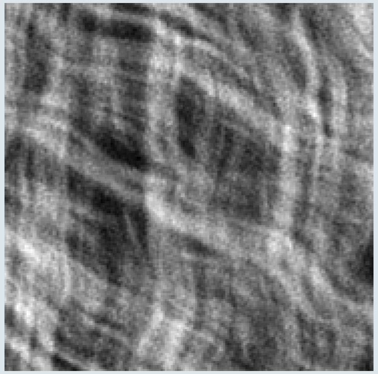
Coherence enhancing diffusion


Size: 128 x 128 x 64

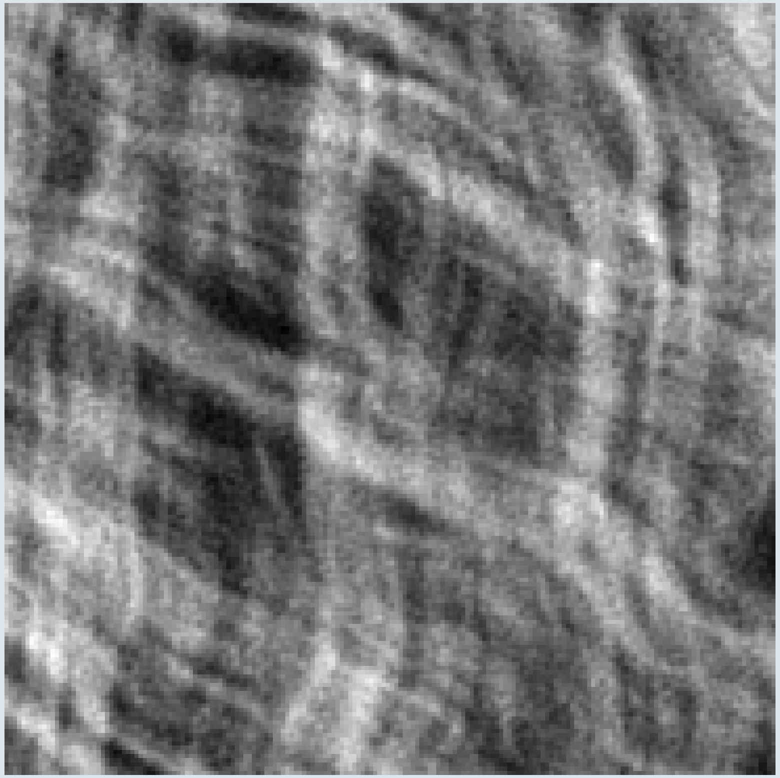
Collagen image



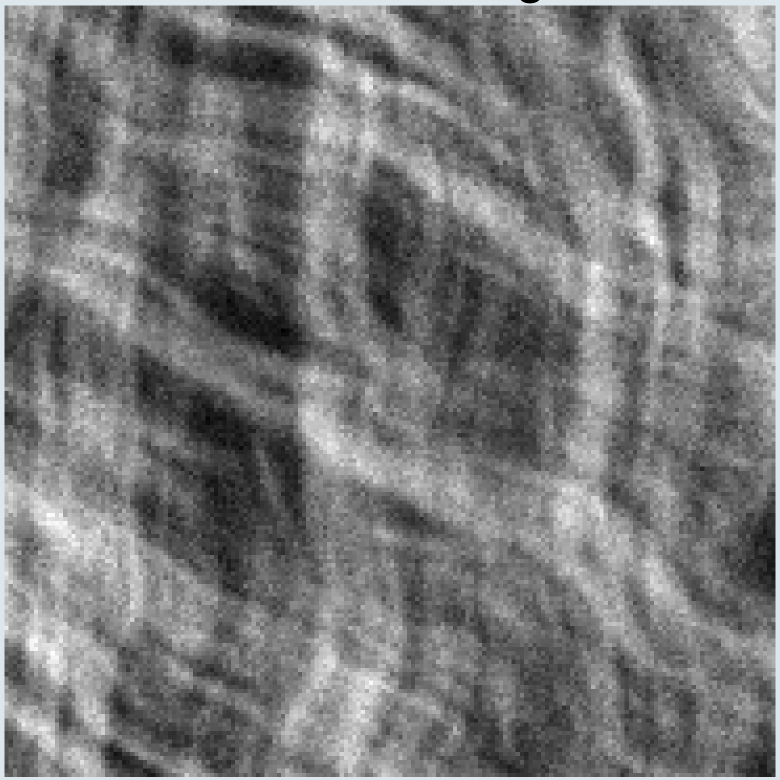
Size: 200 x 200 x 64

$t = 0$

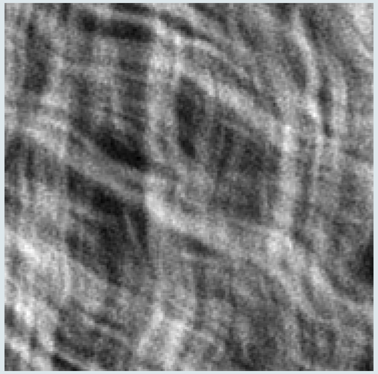
Diffusion in orientation score



Coherence enhancing diffusion



Collagen image



Size: 200 x 200 x 64

$t = 30$

Diffusion in orientation score

Coherence enhancing diffusion

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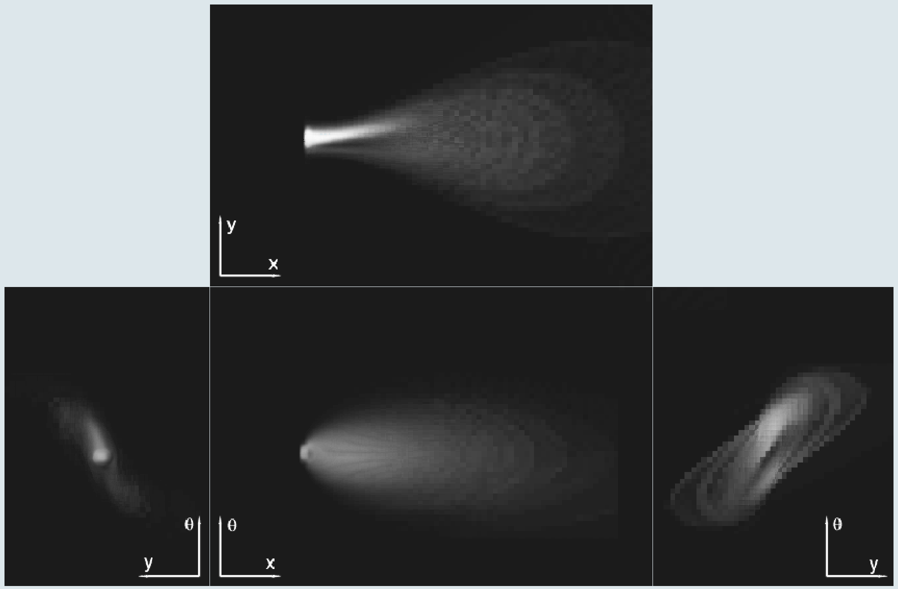
- **Stochastic Completion Fields**
- Using Mathematica

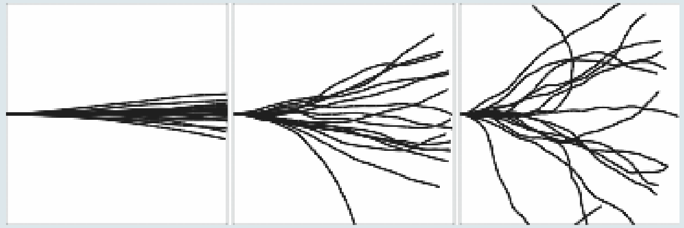
Other PDE: the stochastic completion field

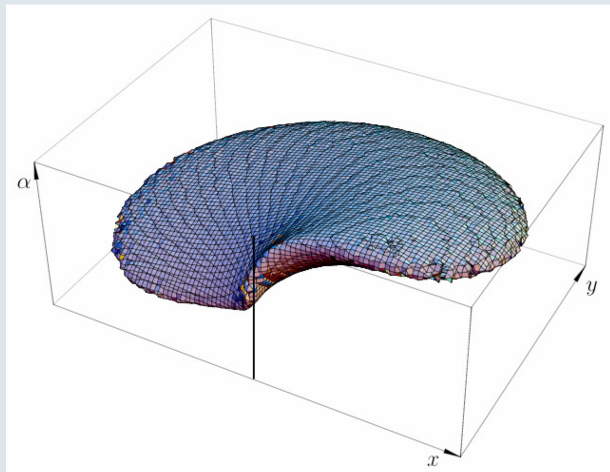
Resolvent of linear PDE

$\partial_t u = (A - \lambda I)u$ with $A = (-\partial_\xi + D_{11}\partial_{\theta\theta})$

It renders probability density field for line continuation based on random walker prior







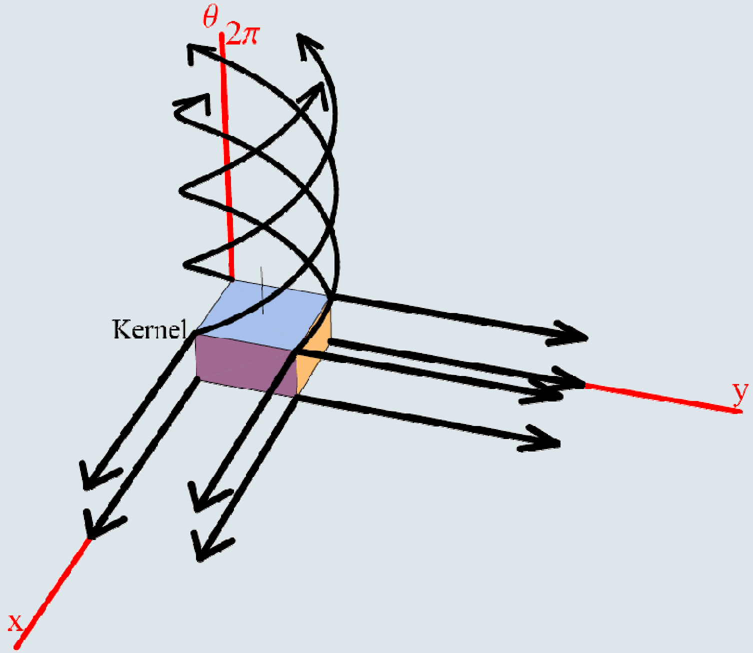
Convolution on Orientation Scores

An image is a function on the translation group

An orientation score is a function on the Euclidean motion group

Normal convolution (on translation group)

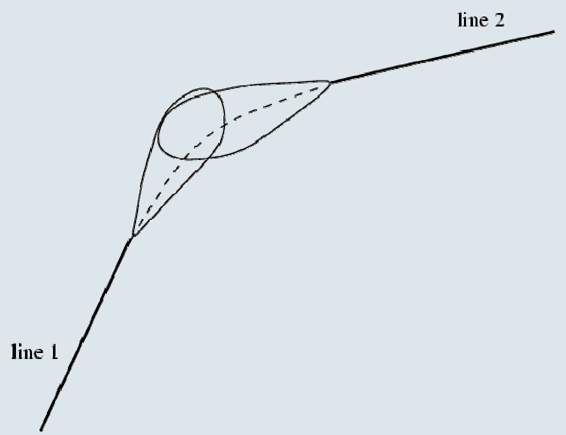
$$[f * g](\mathbf{x}) = \int_{\mathbb{R}^n} f(\mathbf{x}-\mathbf{y})\, g(\mathbf{y})\, d\mathbf{y}$$

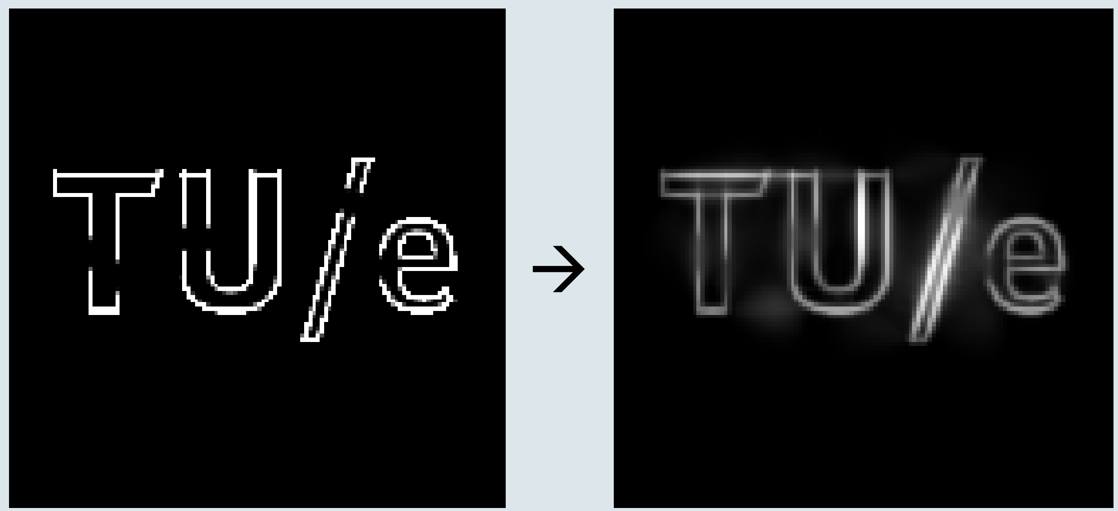


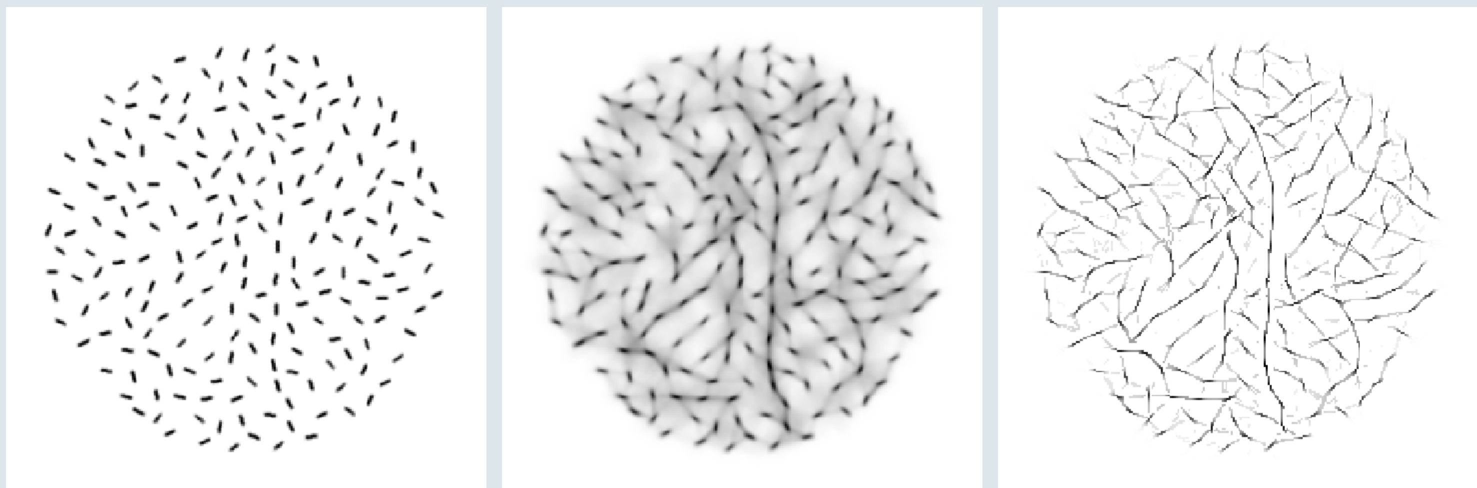
G-convolution, where G is the Euclidean motion group

$$[K *_G U_f](\mathbf{x},\theta) = \int_{\mathbb{R}^2}\int_0^{2\pi} K(R_{\theta'}^{-1}(\mathbf{x}-\mathbf{x}'),\theta-\theta')U_f(\mathbf{x}',\theta')\, d\theta' d\mathbf{x}'$$

Filling Gaps in Curves

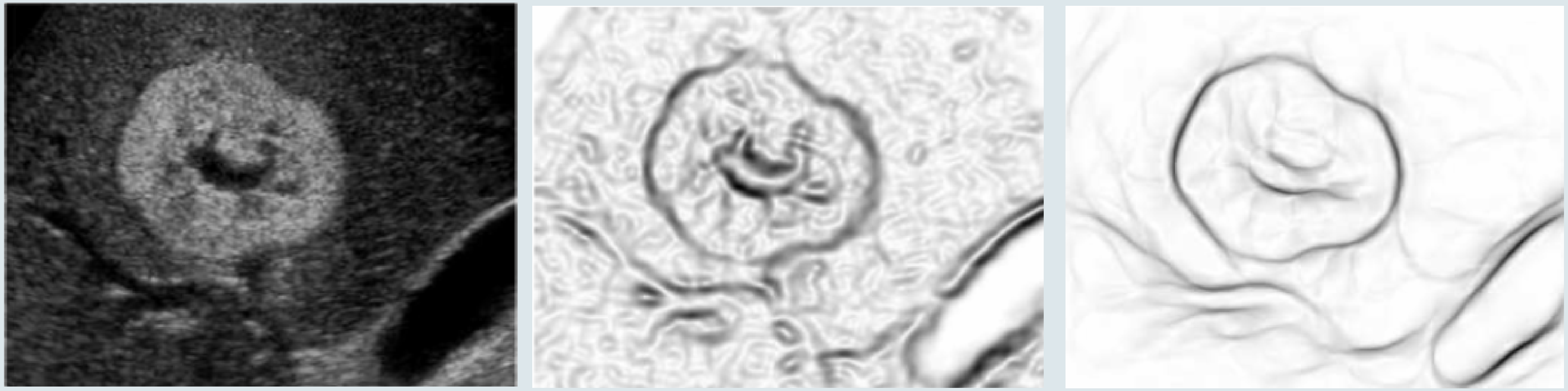




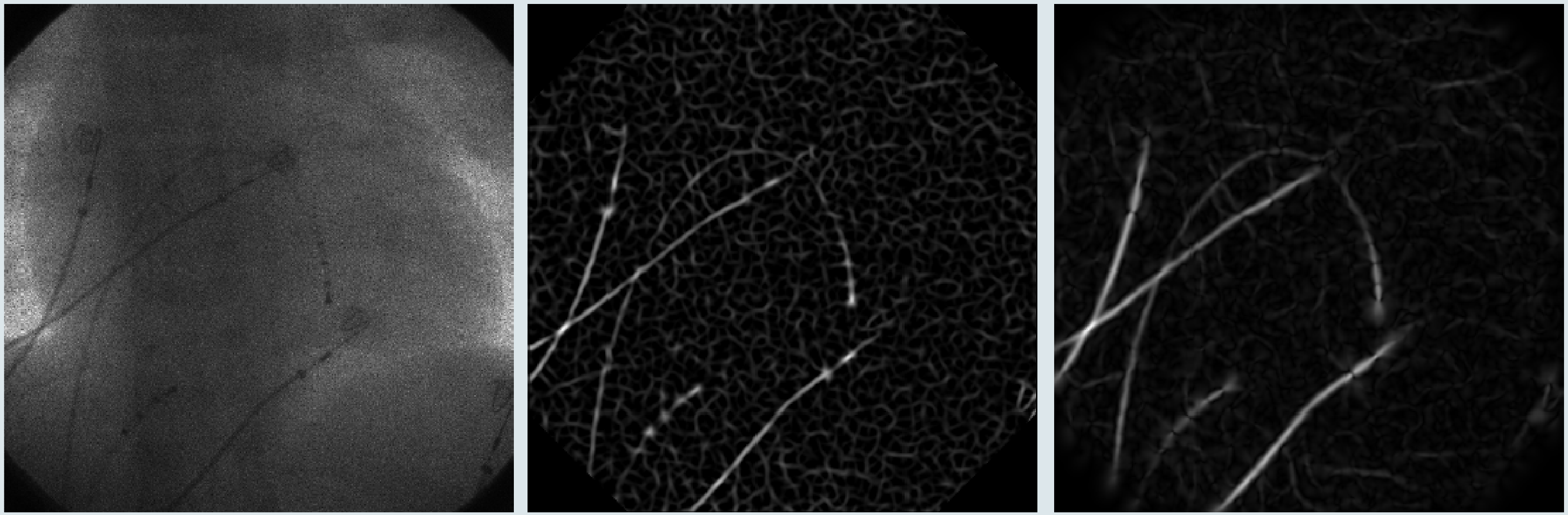


(From M.Sc. thesis by Renske de Boer)

Enhancing edges in Medical images



Source image → Local information → Result



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Using Mathematica

- Mathematica is helpful in solving the math (e.g. non-commuting operators)
- NDSolve in *mathematica* is not usable for our type of PDEs as far as I know
- PDE solver is written in C++, linked with Mathlink
- Typical problems of our PDE
  - Highly anisotropic, not aligned with grid
  - Non-commuting operators
  - Convection + diffusion

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- Remco Duits
- Markus van Almsick
- Bart ter Haar Romeny
- Bart Janssen
- Arjen Ricksen
- Renske de Boer

For questions / more references about this work,
contact e.m.franken@tue.nl