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Medical Image Processing with Orientation Scores

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Department of Biomedical Engineering

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Outline

- **About our Research Group**
- Orientation Scores
- Diffusion in Orientation Scores
- Stochastic Completion Fields
- Using Mathematica

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The Biomedical Image Analysis group

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Mathematica in the BioMIM lab

Starring:
Petr Šereda (Pilsen, Czech Republic)
Tim Peeters (Echt, The Netherlands)

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Our Mathematica Infrastructure

- Full campus license for Mathematica
- The need for "bigmath" kernel servers
  – Bigmath1: Tyan TX46, 4x Opteron 2.2Ghz, 32GB
  – Bigmath2: Tyan VX50, 4x Dualcore Opteron 2.2Ghz, 64GB
  – + 3 older servers
- Use of ParallelMathematica

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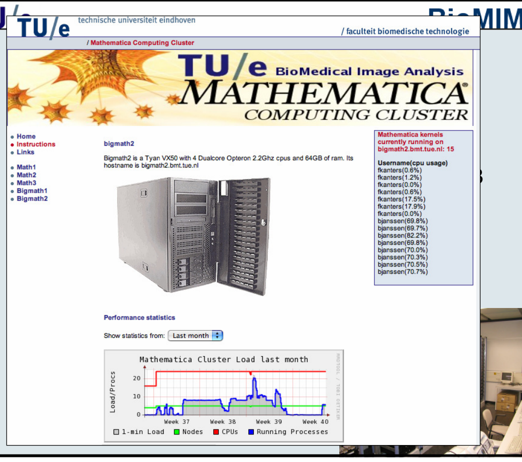

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MathVisionTools

Computer Vision Library for Mathematica:

- Gaussian derivatives
- Geometry driven diffusion
- Orientation score functions
- Image transformations
- DICOM import/export

www.mathvisiontools.net

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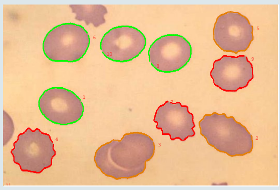
Mathematica in Bachelor Education

Image Analysis for Pathology.

- Groups of 8 2nd year students
- "Invent" image analysis algorithms in *Mathematica*
- Competitive element
- 6 weeks project



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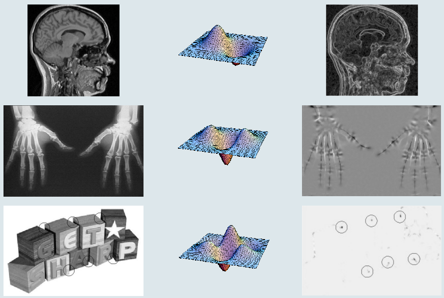
Example: Differential invariants



1st order (edges)

2nd order (ridges)

3rd order (T-junctions)

For example Rotation invariant T-junction detection:

$$\frac{1}{(L_x^2+L_y^2)^3}\left(-L_{xxy}L_y^5 + L_y^4\left(2L_{xy}^2 - L_x(L_{xxx}-2L_{xyy}) + L_{xx}L_{yy}\right) + L_x^4\left(2L_{xy}^2 - L_xL_{xyy} + L_{xx}L_{yy}\right) + L_x^2L_y^2\left(3L_{xx}^2 - 8L_{xy}^2 + L_x(-L_{xxx}+L_{xyy}) - 4L_{xx}L_{yy} + 3L_{yy}^2\right) + L_xL_y^3\left(6L_{xy}(L_{xx}-L_{yy}) + L_x(L_{xxy}-L_{yyy})\right) + L_x^3L_y\left(6L_{xy}(-L_{xx}+L_{yy}) + L_x(2L_{xxy}-L_{yyy})\right)\right)$$

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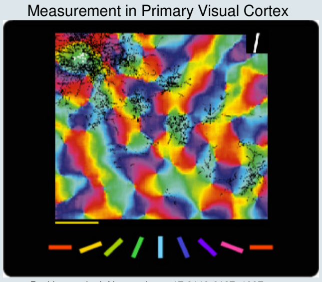
Biological Inspiration

1. The retina contains receptive fields of varying sizes → *multi-scale* sampling device
2. Primary visual cortex is *multi-orientation*

Measurement in Primary Visual Cortex



Bosking et al., J. Neuroscience 17:2112-2127, 1997

- Cells in the primary visual cortex are orientation-specific
- Strong connectivity between cells that respond to (nearly) the same orientation

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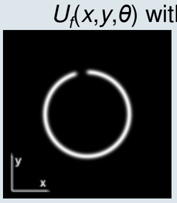
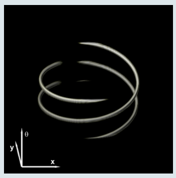
Orientation Scores

From 2D image $f(x,y)$ to orientation score $U_f(x,y,\theta)$ with position (x,y) and orientation θ



$$f(x,y) \xrightarrow{W_\psi} U_f(x,y,\theta)$$

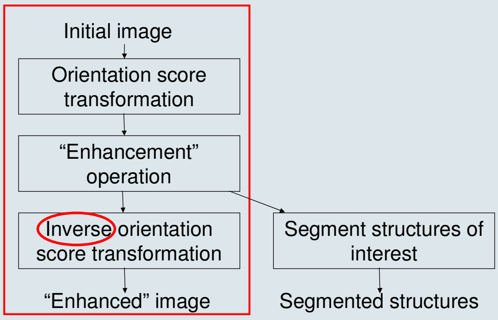


$$(W_\psi[f])(\mathbf{x},\theta) = U_f(\mathbf{x},\theta) = \int_{\mathbb{R}^2} \psi(R_\theta^{-1}(\mathbf{x}'-\mathbf{x}))f(\mathbf{x}')\,d\mathbf{x}'$$

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

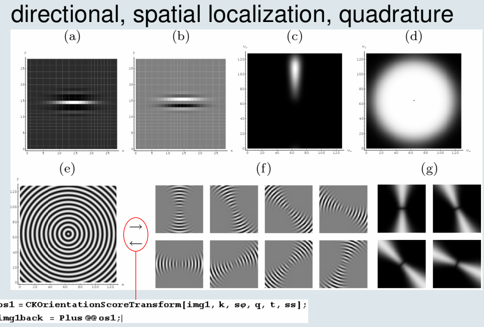
An orientation score is a function on the *Euclidean motion group*

Our approach: Image Processing *via* Orientation Scores



Invertible Orientation Score Transformation

Design considerations: reconstruction, directional, spatial localization, quadrature



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The Diffusion Equation on Images

$$\begin{cases} \frac{\partial}{\partial t} u = \nabla \cdot \mathbf{D} \nabla u \\ u(\mathbf{x}; 0) = f(\mathbf{x}) \end{cases}$$

f = image
u = scale space of image
D = diffusion tensor

$$\nabla = \left(\frac{\partial}{\partial x} \;\; \frac{\partial}{\partial y}\right)^T$$

Linear diffusion	Perona&Malik	Coherence-enhancing diff.
$\mathbf{D} = \mathbf{I}$	$\mathbf{D}(\mathbf{x}) = g(\lvert \nabla u(\mathbf{x}) \rvert)\,\mathbf{I}$	$\mathbf{D}(\mathbf{x}) = s(\mathbf{x})\,\mathbf{v}(\mathbf{x})\,\mathbf{v}^T(\mathbf{x}) + \alpha \mathbf{I}$

$t = 0$

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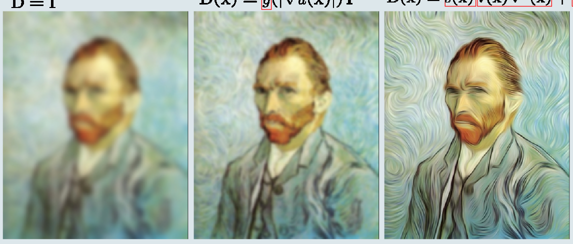


$t = 10$

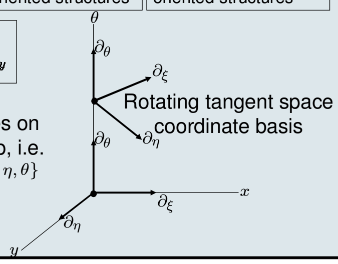
Diffusion in orientation scores

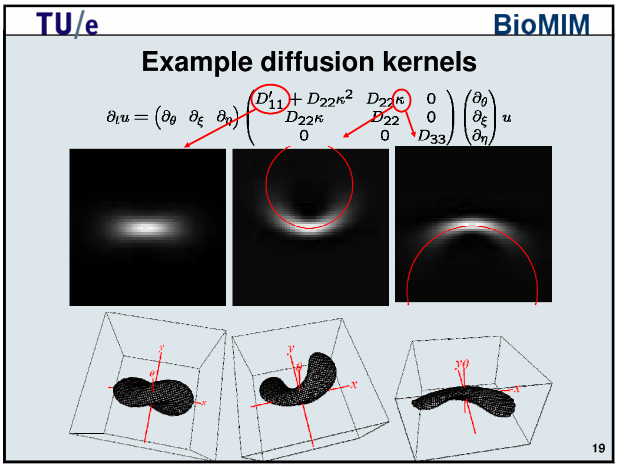
$$\partial_t u = \begin{pmatrix} \partial_\theta & \partial_\xi & \partial_\eta \end{pmatrix} \begin{pmatrix} D'_{11} + D_{22}\kappa^2 & D_{22}\kappa & 0 \\ D_{22}\kappa & D_{22} & 0 \\ 0 & 0 & D_{33} \end{pmatrix} \begin{pmatrix} \partial_\theta \\ \partial_\xi \\ \partial_\eta \end{pmatrix} u$$

curvature; Evolving orientation score; Left-invariant derivatives; Diffusion in orientation; Diffusion tangent to oriented structures; Diffusion orthogonal to oriented structures

$$\partial_\xi = \cos\theta\,\partial_x + \sin\theta\,\partial_y$$
$$\partial_\eta = -\sin\theta\,\partial_x + \cos\theta\,\partial_y$$

∂_ξ, ∂_η, and ∂_θ are left-invariant derivatives on Euclidean motion group, i.e.
$\mathcal{L}_g \partial_i U = \partial_i \mathcal{L}_g U, \;\; i \in \{\xi, \eta, \theta\}$

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Example diffusion kernels
$\partial_t u = (\partial_\theta \ \partial_\xi \ \partial_\eta) \begin{pmatrix} D'_{11} + D_{22}\kappa^2 & D_{22}\kappa & 0 \\ D_{22}\kappa & D_{22} & 0 \\ 0 & 0 & D_{33} \end{pmatrix} \begin{pmatrix} \partial_\theta \\ \partial_\xi \\ \partial_\eta \end{pmatrix} u$
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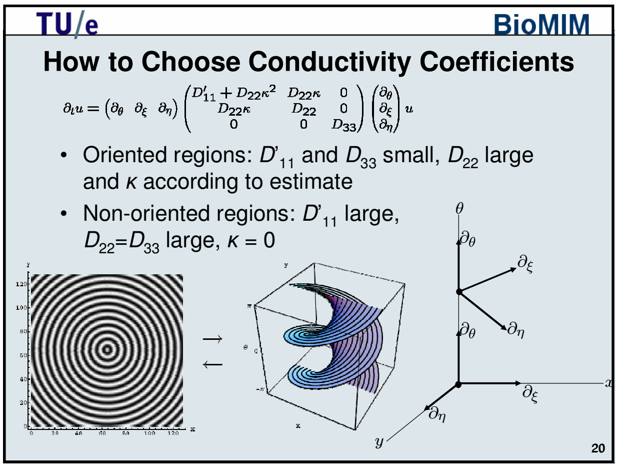
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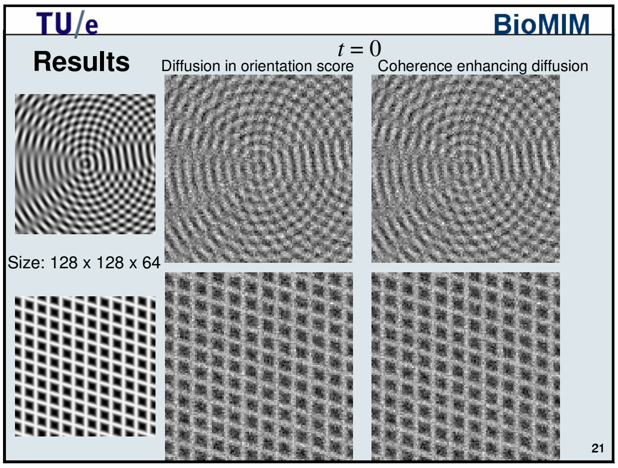
How to Choose Conductivity Coefficients
$\partial_t u = (\partial_\theta \ \partial_\xi \ \partial_\eta) \begin{pmatrix} D'_{11} + D_{22}\kappa^2 & D_{22}\kappa & 0 \\ D_{22}\kappa & D_{22} & 0 \\ 0 & 0 & D_{33} \end{pmatrix} \begin{pmatrix} \partial_\theta \\ \partial_\xi \\ \partial_\eta \end{pmatrix} u$
• Oriented regions: D'_{11} and D_{33} small, D_{22} large and κ according to estimate
• Non-oriented regions: D'_{11} large, D_{22}=D_{33} large, κ = 0
20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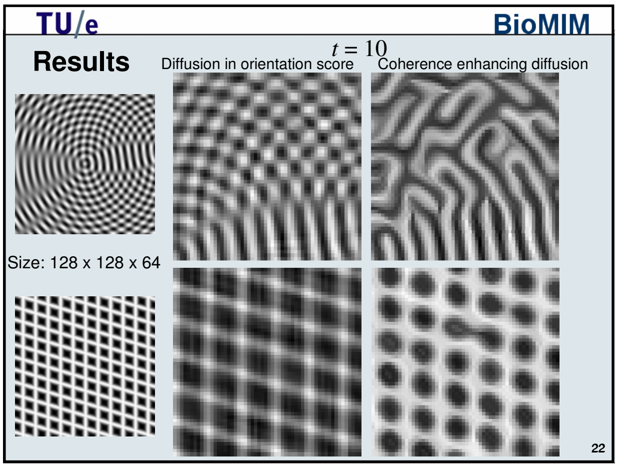
Results
$t = 0$
Diffusion in orientation score
Coherence enhancing diffusion
Size: 128 x 128 x 64
21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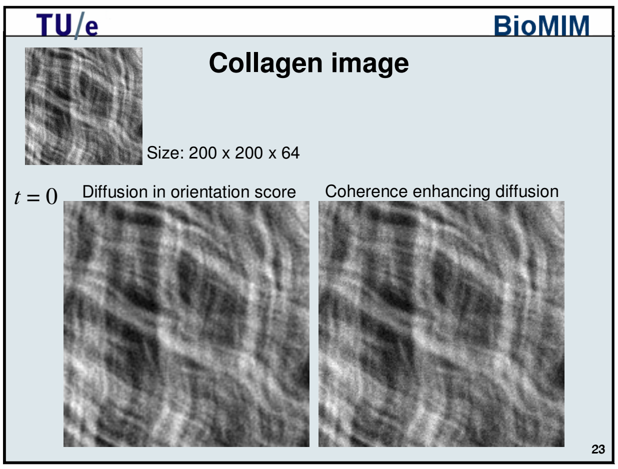
Results
$t = 10$
Diffusion in orientation score
Coherence enhancing diffusion
Size: 128 x 128 x 64
22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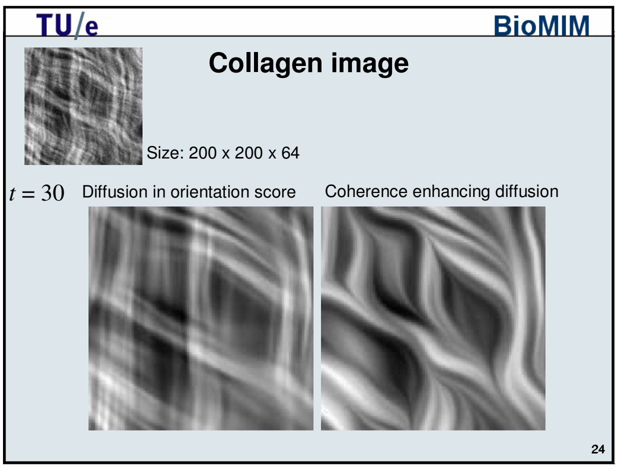
Collagen image
Size: 200 x 200 x 64
$t = 0$
Diffusion in orientation score
Coherence enhancing diffusion
23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BioMIM
Collagen image
Size: 200 x 200 x 64
$t = 30$
Diffusion in orientation score
Coherence enhancing diffusion
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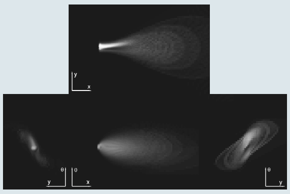
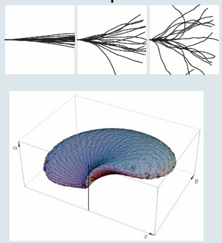
Other PDE: the stochastic completion field

Resolvent of linear PDE
$\partial_t u = (A - \lambda I)u$ with $A = (-\partial_\xi + D_{11}\partial_{\theta\theta})$
It renders probability density field for line continuation based on random walker prior





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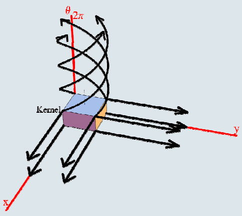
Convolution on Orientation Scores

An image is a function on the translation group

An orientation score is a function on the Euclidean motion group

Normal convolution (on translation group)

$$[f * g](\mathbf{x}) = \int_{\mathbb{R}^n} f(\mathbf{x}-\mathbf{y})\, g(\mathbf{y})\, d\mathbf{y}$$



G-convolution, where G is the Euclidean motion group

$$[K *_G U_f](\mathbf{x},\theta) = \int_{\mathbb{R}^2}\int_0^{2\pi} K(R_{\theta'}^{-1}(\mathbf{x}-\mathbf{x}'), \theta-\theta') U_f(\mathbf{x}',\theta')\, d\theta' d\mathbf{x}'$$

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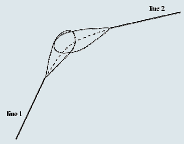
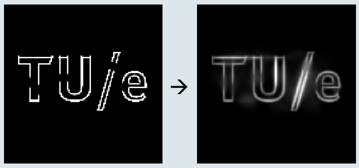
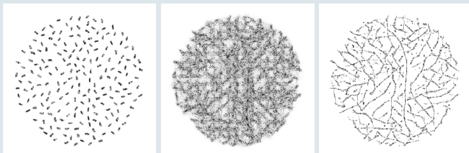
Filling Gaps in Curves







(From M.Sc. thesis by Renske de Boer)

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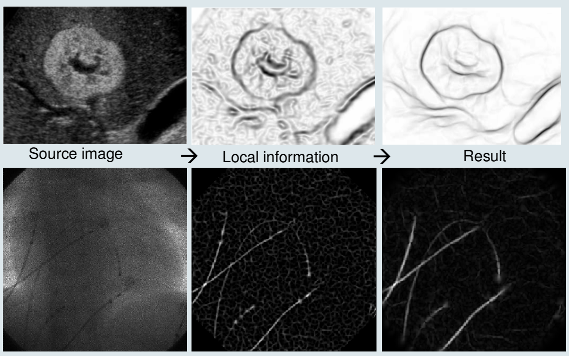
Enhancing edges in Medical images



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Using Mathematica

- Mathematica is helpful in solving the math (e.g. non-commuting operators)
- NDSolve in *mathematica* is not usable for our type of PDEs as far as I know
- PDE solver is written in C++, linked with Mathlink
- Typical problems of our PDE
  - Highly anisotropic, not aligned with grid
  - Non-commuting operators
  - Convection + diffusion

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- Remco Duits
- Markus van Almsick
- Bart ter Haar Romeny
- Bart Janssen
- Arjen Ricksen
- Renske de Boer

For questions / more references about this work, contact e.m.franken@tue.nl