The Potentials of *Mathematica* in Operations Research and Related Applications

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Abstract

This informal article aims at addressing a broad range of *Mathematica* users. A concise review of the quantitative decision-making, modeling and solution paradigm of Operations Research is provided, with an emphasis on using *Mathematica* in this process.

Key Words

Decision-making; quantitative models and algorithmic solution procedures; Operations Research; *Mathematica.*

Quantitative Decision-Making and Optimization Models

In today's competitive global economy, government organizations and private businesses all aim for resource-efficient operations that deliver high quality products and services. This demands prudent, effective and timely decisions and their implementation in an increasingly complex and dynamically changing environment. To illustrate this point, one can think e.g., of decisions related to

- agricultural planning
- distribution systems
- emergency and rescue operations
- engineering systems design
- environmental planning
- financial planning and management
- health care management
- inventory control
- manpower and resource allocation
- manufacturing of goods
- military operations
- production process control
- risk management
- sequencing and scheduling of operations (in various contexts)
- telecommunications
- transportation systems

as well as to other important areas of strategic, tactical, and real-time decision-making.

Finding good quantitative decisions can be frequently viewed and formulated as an *optimization problem*: the decision-maker selects a certain *objective* to maximize (profit, quality, speed of service or job completion, and so on), or to minimize (cost, loss, risk of some undesirable event, etc.). Most typically, a set of (physical, technical, economic, environmental, legal, societal) *constraints* are also considered, when selecting a good *feasible solution* or the *optimal solution*, in the framework of a given problem formulation.

This brief paper reviews the main steps of quantitative decision modeling and problem solution, with a particular emphasis related to using *Mathematica* in this process. A list of illustrative references is also provided.

Operations Research

Operations Research (O.R.) is a comprehensive, scientifically established approach: its practical purpose is to assist analysts and decision-makers in making well-established quantitative decisions.

With more or less emphasis on certain aspects of this verbal 'definition', closely related disciplines have emerged, with significant overlap among them. Systems analysis, management science, control theory, game theory, system simulation, optimization, constraint logic programming, artificial intelligence, fuzzy decision-making, multi-criteria analysis, decision analysis, and so on are all aimed at finding better decisions. (It is outside of the scope of the present discussion to classify these disciplines or their specific approaches, and to discuss their distinctive features.)

One could also mention more 'business-style' phrases such as supply-chain management, enterprise resource planning, total quality management, just-in-time production strategies, materials requirements planning, and others: all these are eventually related to efficient quantitative decision-making.

As many Readers will already know, the name 'Operations Research' is related to the origins of this discipline: a group of British scientists were providing timely advice on military activities during the Second World War. Since then, O.R. has found many other significant applications.

O.R. has witnessed important theoretical advancements in the past (over five) decades. A wide range of sophisticated models and exact – as well as good heuristic – algorithmic solution methods were proposed and rigorously studied. In addition to these theoretical advances, a major factor has been the impact of the 'computer revolution'. Calculations that were considered impossible for centuries became manageable, often on an everyday desktop/portable computer. The overall power of computers has increased by several magnitudes in the last two decades, while they also have become affordable for a broad and ever increasing user base. The synergetic development of mathematical algorithms and computer science has opened up new ways to analyze and handle even the most complex scientific, engineering and economic problems.

Operations Research and its sister disciplines listed above have had an increasing impact on the overall management of complex organizations worldwide. The illustrative references listed at the end of this brief article discuss in detail many case studies that prove this point.

Modeling and Optimization in Operations Research

A formalized procedure aimed at finding optimized decisions includes the following main components:

- Conceptual description of decision problem: abstraction, omission of secondary details and circumstances
- Development of a quantitative model that captures the essential elements of the decision problem, in terms of decision variables, and the relevant functional relationships (constraints and objectives) among these
- Development and/or customization of a systematic (algorithmic) solution procedure to explore the set of feasible decisions, and to select the 'best possible' decision (or perhaps a list of good alternative solutions)
- Numerical solution and its verification; interpretation and summary of results
- Posterior analysis and implementation of the decision(s) found

Note that the main steps outlined above are often carried out in an iterative fashion. These steps are related to working with model versions that gradually refine the model formulation, to the development and testing of a suitable solution procedure, and to the eventual analysis and recommendation. The primary reason for this is that the problems tackled by O.R. or related tools are often complex so that the model components and the solution approach may not be entirely clear from the beginning. One often needs to develop and modify existing models, and to customize algorithmic approaches, to provide an overall meaningful and computationally affordable solution procedure.

Mathematica **for Operations Research**

Mathematica is arguably one of the most ambitious and sophisticated software products ever developed. Its impressive capabilities and range of applications are documented in an increasing library of books and in thousands of professional articles.

A concise list of *Mathematica*'s most significant features includes the following aspects:

- its performance scales well to modeling large, complex problems
- supports rapid prototyping and model/application development
- has a very extensive set of built-in functions
- has impressive symbolic calculation capabilities
- can perform the most advanced computations with high (often arbitrarily high, selectable) precision
- supports several (both high- and lower level, 'traditional') programming styles
- supports elegant, concise coding, and hence easier maintenance and documentation
- includes sophisticated multimedia (visualization, animation, sound,…) tools
- enables advanced technical documentation, desktop publishing, and presentation
- supports links to external software products and to the Internet
- portable across a broad range of hardware platforms and operating systems.

In summary, *Mathematica* is a fully integrated scientific and technical modeling and computing environment that, in many aspects, provides a 'one-stop' solution.

In the Operations Research context, a particular emphasis may be placed on several of the above points, such as overall performance, rapid prototyping, advanced computations, concise code development, external link options, and the possibility of integrated documentation in 'live' *Mathematica* notebooks. These features have an important role in the stages of O.R. model formulation and verification, algorithm development, numerical solution and analysis, and project documentation.

It is also worth of emphasizing that, although *all* interpreted computational environments (starting from Excel, through optimization-specific languages to other more general-purpose computing systems) have a relatively slower program execution speed when compared to a compiled 'number crunching' solver system, the overall application development time (manpower needs) can often be massively reduced. (It is most instructive to recall in this context the debate that surrounded the early development of programming languages – such as Fortran, Algol, Basic, Pascal, etc. – as opposed to machine-level assembly programming.)

Application Perspectives

We see particularly strong application potentials for *Mathematica* in Operations Research, when the decision model can not be brought to one of the most simple 'standard' forms – notably, continuous linear programming and its 'simplest' extensions. Model simplicity is a virtue, of course, unless it leads to unjustifiable over-simplification. While one should not make the claim that 'the world is [only] nonlinear' (such claim was actually made in a historical setting, when Dantzig introduced linear programming to a learned audience), nobody should claim either that complex nonlinearities are a mere exception in the real world… The amazing variety of natural forms and processes, as well as all efforts to manage such systems prove otherwise.

A large variety of advanced O.R. concepts and tools – such as nonlinear and stochastic programming, combinatorial and mixed integer (linear/nonlinear) optimization, and suitable methods to solve these models – certainly belong to the scope of problems that can be successfully analyzed and solved using *Mathematica*. One of the primary reasons for this is that such models may not be easy (or possible) to cast into a pre-defined, 'boxed' formulation which then can be simply submitted to a 'number-crunching' executable solver. To illustrate this point, one can think of optimization models that may be made up by, in principle, *arbitrary* continuous nonlinear functions (that may describe the operations of a complex engineering or environmental system). In such cases, modeling and code development are both essential, and using *Mathematica* as the primary development platform often proves to be a very good choice.

The class of such advanced O.R. models is relevant and broad, and recent results in combinatorial/nonlinear/global/stochastic modeling and optimization will lead to their increased consideration and usage in solving complex real-world issues. It should be noted at this point that the repertoire and quality of *Mathematica* functions and third-party developer application packages to solve various optimization models has been steadily increasing in recent years.

This brief discussion is concluded by a short, generic list of application areas that can greatly profit from a modeling and solution approach that relies on using *Mathematica*. Three very broad classes of such applications are from the areas of

- optimization of complex 'black box' systems
- optimal control of dynamic systems
- decision-making under uncertainty

Such models are ubiquitous not only in many areas of applied mathematics, but also in physical, chemical, biochemical, environmental, pharmaceutical, medical, financial, and industrial research, and in the related industries.

A few more concrete examples from the author's own practice (with clients and co-authors):

- cancer therapy design
- chemical systems and process modeling
- data analysis, classification and visualization
- economic and financial forecasting
- environmental risk assessment and management
- industrial design
- laser equipment design
- model fitting to data (calibration)
- optimal operation of 'closed' (confidential) systems
- packing and other object arrangement design problems
- robot design and manipulations
- sonar equipment design
- systems of nonlinear equations and inequalities
- wastewater treatment systems management
- routing and scheduling problems

It is noted that several of these model-types were in, fact, analyzed and solved using *Mathematica* – and that most likely all could have been...

Further prospective and actual application areas and examples are welcome: please feel free to contact the author.

Illustrative References

There are literally thousands of books published worldwide that are all related to quantitative modeling, Operations Research, and *Mathematica*. The following list is merely an illustrative sample of works that discuss general scientific, *Mathematica*-based or Operations Research modeling and/or optimization topics.

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About the Author

János D. Pintér, MSc, PhD, DSc is a researcher and practitioner with three decades of professional experience. His main area of interest is advanced modeling and optimization, algorithm and software development. He wrote and edited several books, and over 140 articles and technical reports. The software products developed by him (marketed through his company) have been in use by academic, government and business organizations in some 15 countries. Two of these products, *MathOptimizer* and *MathOptimizer Professional* have been developed for use with *Mathematica*.

Dr. Pintér is a member of INFORMS, SIAM, and of the Canadian and Hungarian O.R. Societies. He serves on the Editorial Board of the *Journal of Global Optimization*, the *Journal of Applied Mathematics and Decision Sciences*, and of two on-line journals maintained by the GAMS Corporation. He received the 2000 INFORMS Computing Society Prize for his book *Global Optimization in Action*, as well as other research awards and fellowships (e.g., in Australia, Austria, Canada, Germany, Hungary, Italy, Netherlands, and the United States). Dr. Pintér has been presenting lectures, tutorials, and workshops in over 25 countries.

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