Mathematics of Relativity And Its Application To Some Computations

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A new system of observer-dependent arithmetic on finite sets of finite decimal fractions is developed. Given the system, various results in algebra and geometry are discussed. Further, the system's application to special physical theory of relativity is considered. In particular, a model of coordinate transformations is created.

Arithmetic

- The sets W_n

- Addition and Subtraction

$$c \pm_n d = \begin{cases} c \pm d \text{ if } c \pm d \in W_n \\ \text{not defined if } c \pm d \notin W_n \end{cases}$$

- Multiplication

$$c \times_n d = \sum_{k=0}^{n} {}^{n} \sum_{m=0}^{n-k} {}^{n} 0.\underbrace{0\ldots0}_{k-1} c_k \cdot 0.\underbrace{0\ldots0}_{m-1} d_m$$

- Division

$$c \div_n d = \begin{cases} r \text{ if } \exists!\ r \in W_n \quad r \times_n d = c \\ \text{not def. if no } r \text{ exists/not unique} \end{cases}$$

Algebra

1. Equation solving

2. Dimension

Geometry

Intersection of Lines

- One point:

$$\begin{cases} (0.59 \times_2 x +_2 0.79 \times_2 y) +_2 0.59 = 0 \\ (1.00 \times_2 x +_2 1.00 \times_2 y) +_2 0.41 = 0 \end{cases}$$

- Two points:

$$\begin{cases} (0.31 \times_2 x +_2 1.00 \times_2 y) +_2 0.63 = 0 \\ (1.00 \times_2 x +_2 0.34 \times_2 y) +_2 0.91 = 0 \end{cases}$$

- Ten points:

$$\begin{cases} (0.08 \times_2 x +_2 0.78 \times_2 y) +_2 0.09 = 0 \\ (-0.47 \times_2 x -_2 0.75 \times_2 y) -_2 0.38 = 0 \end{cases}$$

- One hundred points:

$$\begin{cases} (0.14 \times_2 x +_2 0.23 \times_2 y) -_2 0.22 = 0 \\ (0.61 \times_2 x +_2 0.43 \times_2 y) -_2 0.76 = 0 \end{cases}$$

Euclidean Vs Lobachevsky

Example on the Chalkboard

Coordinate Transformations

- Classical Setting

  Point on a two-dimensional manifold with the coordinates (x, t) in K and (x', t') in K', has the following properties: $x -_n c \times_n t = 0$ iff $x' -_n c \times_n t' = 0$ and $x +_n c \times_n t = 0$ iff $x' +_n c \times_n t' = 0$.

  These relationships are given by

  $x - ct = \lambda\,(x' - ct')$ and $x + ct = \mu\,(x' + ct')$.

- W_n setting

  The above equations are invalid. For example, if $\lambda < 1$ and $x -_n c \times_n t = 0.\underbrace{0...0}_{n-1}1$, then $x' -_n c \times_n t' = 0$. Similarly, if $\mu < 1$ and $x +_n c \times_n t = 0.\underbrace{0...0}_{n-1}1$, then $x' +_n c \times_n t' = 0$.

Proposed Solution

$$\begin{cases} \lambda_1 \times_n (x -_n c \times_n t) +_n \lambda_2 \times_n (x' -_n c \times_n t') = 0 \\ \mu_1 \times_n (x +_n c \times_n t) +_n \mu_2 \times_n (x' +_n c \times_n t') = 0 \end{cases}$$

- Need to find $\lambda_1, \lambda_2, \mu_1, \mu_2$
- Assume that $(\lambda_1, \lambda_2) = 1$ and $(\mu_1, \mu_2) = 1$

Solving for Coefficients

- Coordinates of O' are $x' = 0$ in K' and $x = v \times_n t$ in K

$$\begin{cases} \lambda_1 \times_n (v \times_n t -_n c \times_n t) +_n \lambda_2 \times_n (c \times_n t') = 0 \\ \mu_1 \times_n (v \times_n t +_n c \times_n t) +_n \mu_2 \times_n (c \times_n t') = 0 \end{cases}$$

- Let $t = 0$

$$\begin{cases} \lambda_1 \times_n (x) +_n \lambda_2 \times_n (x' -_n c \times_n t') = 0 \\ \mu_1 \times_n (x) +_n \mu_2 \times_n (x' +_n c \times_n t') = 0 \end{cases}$$

- Now we can compute

$$x' = f_1 (x, \lambda_1, \lambda_2, \mu_1, \mu_2)$$

excluding t' from calculations.

Solving for Coefficients (Cont'd)

- Set $x' = 1$, we can compute

$$x = \Delta x = g_1 \left(\lambda_1, \lambda_2, \mu_1, \mu_2\right)$$

- Let $t' = 0$

$$\begin{cases} \lambda_1 \times_n \left(x -_n c \times_n t\right) +_n \lambda_2 \times_n \left(x'\right) = 0 \\ \mu_1 \times_n \left(x +_n c \times_n t\right) +_n \mu_2 \times_n \left(x'\right) = 0 \end{cases}$$

- Compute

$$x = f_2 \left(x', \lambda_1, \lambda_2, \mu_1, \mu_2\right)$$

  excluding t from calculations.

- Set $x = 1$ and compute

$$x' = \Delta x' = g_2 \left(\lambda_1, \lambda_2, \mu_1, \mu_2\right)$$

- Therefore, we have

$$\Delta x = \Delta x'$$

which gives us a W_n model of coordinate transformations.