Preface

This book provides an introduction to the theory of dynamical systems with the aid of the Mathematica® computer algebra system. It is written for both senior undergraduates and graduate students. The first part of the book deals with continuous systems using ordinary differential equations (Chapters 1–10), the second part is devoted to the study of discrete dynamical systems (Chapters 11–15), and Chapters 16 and 17 deal with both continuous and discrete systems. It should be pointed out that dynamical systems theory is not limited to these topics but also encompasses partial differential equations, integral and integro-differential equations, stochastic systems and time delay systems, for instance. References [1]–[4] given at the end of the Preface provide more information for the interested reader. The author has gone for breadth of coverage rather than fine detail and theorems with proof are kept at a minimum. The material is not clouded by functional analytic and group theoretical definitions, and so is intelligible to readers with a general mathematical background. Some of the topics covered are scarcely covered elsewhere. Most of the material in Chapters 9, 10, 14, 16 and 17 is at postgraduate level and has been influenced by the author's own research interests. There is more theory in these chapters than in the rest of the book since it is not easily accessed anywhere else. It has been found that these chapters are especially useful as reference material for senior undergraduate project work. The theory in other chapters of the book is dealt with more comprehensively in other texts, some of which may be found in the references section of the corresponding chapter. The book has a very hands-on approach and takes the reader from the basic theory right through to recently published research material.

Mathematica is extremely popular with a wide range of researchers from all sorts of disciplines. It is a symbolic, numerical and graphical manipulation package which makes it ideal for the study of nonlinear dynamical systems.

Chapter 0 provides an introduction to the high-level computer language Mathematica, developed by Wolfram Research. The reader is shown how to use both text based input commands and palettes. Students should be able to complete tutorials one and two in under two hours depending upon their past experience. New users will find that the tutorials enable them to become familiar with Mathematica within a few hours. Both engineering and mathematics students appreciate this method of teaching, and the author has found that it generally works well with a ratio of one staff member to about 20 students in a computer laboratory. Those moderately familiar with the package and even the expert users will find Chapter 0 to be a useful source of reference. The Manipulate command along with some simple Mathematica programs with output are introduced in the next section. Mathematica program files in the rest of the book are listed at the end of each chapter to avoid unnecessary cluttering in the text. The author suggests that the reader should save the relevant example programs listed throughout the book in separate notebooks. These programs can then be edited accordingly when attempting the exercises at the end of each chapter. The Mathematica commands, notebooks, programs and output can also be viewed in color over the Web at Mathematica's Information Center

http://library.wolfram.com/infocenter/Books/Mathematics/.

The Mathematica program files can also be downloaded at this site. Throughout this book, Mathematica is viewed as a tool for solving systems or producing eye-catching graphics. The author has used Mathematica 6.0 in the preparation of the material. However, the Mathematica programs have been kept as simple as possible and should also run under later versions of the package. One of the advantages of using the Information Center rather than an attached CD is that programs can be updated as new versions of Mathematica are released.

The first few chapters of the book cover some theory of ordinary differential equations and applications to models in the real world are given. The theory of differential equations applied to chemical kinetics and electric circuits is introduced in some detail. Chapter 1 ends with the existence and uniqueness theorem for the solutions of certain types of differential equation. A variety of numerical procedures are available in Mathematica when solving stiff and nonstiff systems when an analytic solution does not exist or is extremely difficult to find. The theory behind the construction of phase plane portraits for two-dimensional systems is dealt with in Chapter 2. Applications are taken from chemical kinetics, economics, electronics, epidemiology, mechanics, population dynamics; and modeling the populations of interacting species are discussed in some detail in Chapter 3. Limit cycles, or isolated periodic solutions, are introduced in Chapter 4. Since we live in a periodic world, these are the most common type of solution found when modeling nonlinear dynamical systems. They appear extensively when modeling both the technological and natural sciences. Hamiltonian, or conservative, systems and stability are discussed in Chapter 5, and Chapter 6 is concerned with how planar systems vary depending upon a parameter. Bifurcation, bistability, multistability, and normal forms are discussed.

The concept of chaos is expanded upon in Chapters 7 and 8, where three-dimensional systems and Poincaré maps are investigated. These higher dimensional systems can exhibit strange attractors and chaotic dynamics. One can plot the three-dimensional objects in Mathematica and graph time series plots to get a better understanding of the dynamics involved. Once again the theory can be applied to chemical kinetics (including stiff systems), electric circuits, and epidemiology; a simplified model for the weather is also briefly discussed. The next chapter deals with Poincaré first return maps that can be used to untangle complicated interlacing trajectories in higher-dimensional spaces. A periodically driven nonlinear pendulum is also investigated by means of a nonautonomous differential equation. Both local and global bifurcations are investigated in Chapter 9. The main results and statement of the famous second part of David Hilbert's sixteenth problem are listed in Chapter 10. In order to understand these results, Poincaré compactification is introduced. The study of continuous systems ends with one of the authors specialities—limit cycles of Liénard systems. There is some detail on Liénard systems in particular in this part of the book, but they do have a ubiquity for systems in the plane.

Chapters 11–15 deal with discrete dynamical systems. Chapter 11 starts with a general introduction to iteration and linear recurrence (or difference) equations. The bulk of the chapter is concerned with the Leslie model used to investigate the population of a single species split into different age classes. Harvesting and culling policies are then investigated and optimal solutions are sought. Nonlinear discrete dynamical systems are dealt with in Chapter 12. Bifurcation diagrams, chaos, intermittency, Lyapunov exponents, periodicity, quasiperiodicity, and universality are some of the topics discussed. The theory is then applied to real-world problems from a broad range of disciplines including population dynamics, biology, economics, nonlinear optics, and neural networks. The next chapter is concerned with complex iterative maps, Julia sets and the now famous Mandelbrot set are plotted. Basins of attraction are investigated for the first time in this text. As a simple introduction to optics, electromagnetic waves and Maxwell's equations are studied at the beginning of

Chapter 14. Complex iterative equations are used to model the propagation of light waves through nonlinear optical fibers. A brief history of nonlinear bistable optical resonators is discussed and the simple fibre ring resonator is analyzed in particular. Chapter 14 is devoted to the study of these optical resonators and phenomena such as bistability, chaotic attractors, feedback, hysteresis, instability, linear stability analysis, multistability, nonlinearity, and steady-states are dealt with. The first and second iterative methods are defined in this chapter. Some simple fractals may be constructed using pencil and paper in Chapter 15, and the concept of fractal dimension is introduced. Fractals may be thought of as identical motifs repeated on ever reduced scales. Unfortunately, most of the fractals appearing in nature are not homogeneous but are more heterogeneous, hence the need for the multifractal theory given later in the chapter. It has been found that the distribution of stars and galaxies in our universe are multifractal, and there is even evidence of multifractals in rainfall, stock markets, and heartbeat rhythms. Applications in materials science, geoscience, and image processing are briefly discussed.

The next chapter is devoted to the new and exciting theory behind chaos control and synchronization. For most systems, the maxim used by engineers in the past has been "stability good, chaos bad", but more and more nowadays this is being replaced with "stability good, chaos better". There are exciting and novel applications in cardiology, communications, engineering, laser technology, and space research, for example.

A brief introduction to the enticing field of neural networks is presented in Chapter 17. Imagine trying to make a computer mimic the human brain. One could ask the question: In the future will it be possible for computers to think and even be conscious? The human brain will always be more powerful than traditional, sequential, logic-based digital computers and scientists are trying to incorporate some features of the brain into modern computing. Neural networks perform through learning and no underlying equations are required. Mathematicians and computer scientists are attempting to mimic the way neurons work together via synapses, indeed, a neural network can be thought of as a crude multidimensional model of the human brain. The potential for this theory is still largely unexplored, but the expectations are high for future applications in a broad range of disciplines. Neural networks are already being used in pattern recognition (credit card fraud, prediction and forecasting, disease recognition, facial and speech recognition), psychological profiling, predicting wave overtopping events, and control problems, for example. They also provide a parallel architecture allowing for very fast computational and response times. In recent years, the disciplines of neural networks and nonlinear dynamics have increasingly coalesced and a new branch of science called neurodynamics is emerging. Lyapunov functions can be used to determine the stability of certain types of neural network. There is also evidence of chaos, feedback, nonlinearity, periodicity, and chaos synchronization in the brain.

Chapter 18 lists examination-type questions; the first section to be used without the package and the second section to be used with the Mathematica package in a computer laboratory.

Both textbooks and research papers are presented in the list of references. The textbooks can be used to gain more background material, and the research papers have been given to encourage further reading and independent study.

This book is informed by the research interests of the author which are currently nonlinear ordinary differential equations, nonlinear optics, multifractals, and neural networks. Some references include recently published research articles by the author.

The prerequisites for studying dynamical systems using this book are undergraduate courses in linear algebra, real and complex analysis, calculus and ordinary differential equations; a knowledge of a computer language such as C or Fortran would be beneficial but not essential.

Recommended Textbooks

[1] B. Bhattacharya and M. Majumdar, *Random Dynamical Systems: Theory and Applications*, Cambridge University Press, 2007.

[2] J. Chiasson, and J.J. Loiseau, *Applications of Time Delay Systems*, Springer, 2007.

[3] V. Volterra, *Theory of Functionals and of Integral and Integro-Differential Equations*, Dover Publications, 2005.

[4] J.K. Hale, L.T. Magalhaes and W. Oliva, *Dynamics in Infinite Dimensions*, Springer, 2nd ed., 2002.

I would like to express my sincere thanks to Wolfram Research for supplying me with the latest versions of Mathematica. Thanks also go to all of the reviewers from the first editions of the Maple and MATLAB books. Special thanks go to Tom Grasso and Ann Kostant (Executive Editor, Mathematics and Physics, Birkhäuser). Thanks to the referees of the first draft of this book for their useful comments and suggestions. Finally, thanks to my family and especially my wife Gaynor, and our children, Sebastian and Thalia, for their continuing love, inspiration and support.

Stephen Lynch