DYNAMIC INTERACTIVITY

۲

Wolfram Mathematica® Tutorial Collection

۲

For use with Wolfram *Mathematica*[®] 7.0 and later.

For the latest updates and corrections to this manual: visit reference.wolfram.com

For information on additional copies of this documentation: visit the Customer Service website at www.wolfram.com/services/customerservice or email Customer Service at info@wolfram.com

Comments on this manual are welcomed at: comments@wolfram.com

Content authored by: Theodore Gray and Lou D'Andria

Printed in the United States of America. 15 14 13 12 11 10 9 8 7 6 5 4 3 2

©2008 Wolfram Research, Inc.

All rights reserved. No part of this document may be reproduced or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior written permission of the copyright holder.

Wolfram Research is the holder of the copyright to the Wolfram *Mathematica* software system ("Software") described in this document, including without limitation such aspects of the system as its code, structure, sequence, organization, "look and feel," programming language, and compilation of command names. Use of the Software unless pursuant to the terms of a license granted by Wolfram Research or as otherwise authorized by law is an infringement of the copyright.

Wolfram Research, Inc. and Wolfram Media, Inc. ("Wolfram") make no representations, express, statutory, or implied, with respect to the Software (or any aspect thereof), including, without limitation, any implied warranties of merchantability, interoperability, or fitness for a particular purpose, all of which are expressly disclaimed. Wolfram does not warrant that the functions of the Software will meet your requirements or that the operation of the Software will be uninterrupted or error free. As such, Wolfram does not recommend the use of the software described in this document for applications in which errors or omissions could threaten life, injury or significant loss.

Mathematica, MathLink, and MathSource are registered trademarks of Wolfram Research, Inc. J/Link, MathLM, .NET/Link, and webMathematica are trademarks of Wolfram Research, Inc. Windows is a registered trademark of Microsoft Corporation in the United States and other countries. Macintosh is a registered trademark of Apple Computer, Inc. All other trademarks used herein are the property of their respective owners. Mathematica is not associated with Mathematica Policy Research, Inc.

Contents

Introduction to Dynamic	1
Advanced Dynamic Functionality	20
Introduction to Manipulate	44
Advanced Manipulate Functionality 1	.04
Generalized Input 1	.26
Views	41

Introduction to Dynamic

This tutorial describes the principles behind Dynamic, DynamicModule and related functions, and goes into detail about how they interact with each other and with the rest of *Mathematica*.

These functions are the foundation of a higher-level function Manipulate that provides a simple yet powerful way of creating a great many interactive examples, programs, and demonstrations, all in a very convenient, though relatively rigid, structure. If that structure solves the problem at hand, you need look no further than Manipulate and you do not need to read this tutorial. However, do continue with this tutorial if you want to build a wider range of structures, including complex user interfaces.

This is a hands-on tutorial. You are expected to evaluate all the input lines as you reach them and watch what happens. The accompanying text will not make sense without evaluating as you read.

The Fundamental Principle of Dynamic

Ordinary *Mathematica* sessions consist of a series of static inputs and outputs, which form a record of calculations done in the order in which they were entered.

Evaluate each of these four input cells one after the other.

```
In[1]:= x = 5;
In[2]:= x<sup>2</sup>
Out[2]= 25
In[1]:= x = 7;
In[4]:= x<sup>2</sup>
Out[4]= 49
```

The first output still shows the value from when x was 5, even though it is now 7. This is, of course, very useful, if you want to see a history of what you have been doing. However, you may often want a fundamentally different kind of output, one that is automatically updated to always reflect its current value. This new kind of output is provided by Dynamic.

Evaluate the following cell; note that the result will be 49 because the current value of x is 7.

In[5]:= **Dynamic** $[x^2]$

In fact it is generally the case that when you first evaluate an input that contains variables wrapped in Dynamic, you will get the same result as you would have without Dynamic. But if you subsequently change the value of the variables, the displayed output will change retroactively.

Evaluate the following cells one at a time, and note the change in the value displayed above.

In[6]:= x = 9; In[7]:= x = 15; In[8]:= x = 10;

The first two static outputs are still 25 and 49 respectively, but the single dynamic output now displays 100, the square of the last value of x. (This sentence will, of course, become incorrect as soon as the value of x is changed again.)

There are no restrictions on the kinds of values that can go into a dynamic output. Just because *x* was initially a number does not mean it cannot become a formula or even a graphic in subsequent evaluations. This might seem like a simple feature, but it is the basis for a very powerful set of interactive capabilities.

Each time the value of x is changed, the dynamic output above is updated automatically. (You might need to scroll back to see it.)

```
In[9]:= x = Integrate \left[\frac{1}{1-y^{3}}, y\right];
In[10]:= x = Plot[Sin[x], \{x, 0, 2Pi\}];
In[1]:= x = 0;
```

Dynamic [expr]

an object that displays as the dynamically updated current value of *expr*

Basic dynamic expression.

Dynamic and Controls

Dynamic is often used in connection with controls such as sliders and checkboxes. The full range of controls available in *Mathematica* is discussed in "Control Objects"; here sliders are used to illustrate how things work. The principles of using Dynamic with other controls is basically the same.

A slider is created by evaluating the slider function, in which the first argument is the position and the optional second argument specifies the range and step size, with the default range from 0 to 1 and the default step size 0.

This is a slider in a centered position.



Click on the thumb and move it around. The thumb moves, but nothing else happens since the slider is not connected to anything.

This associates the position of the slider with the current value of the variable x. (This form is explained in more detail later.)

```
In[12]:= Slider[Dynamic[x]]
```

Out[13]=

This creates a new dynamic output of x since the last one has probably scrolled off your screen by now.

```
In[14]:= Dynamic[x]
```

Out[14]= **0.**

Drag the last slider around. As the slider moves, the value of x changes and the dynamic output updates in real time.

The slider also responds to changes in the value of *x*.

To see this, evaluate this line. In[15]:= x = 0.8; You should see the slider jump, and the dynamic output of *x* change, simultaneously.

	This creates another <i>x</i> slider.
In[16]:=	<pre>Slider[Dynamic[x]]</pre>
Out[16]=	-]

Notice that if you move *either* of the two sliders you now have, the other one moves in "lock sync." Both are connected, dynamically and bi-directionally, to the current value of *x*.

Dynamic and Other Functions

Dynamic and control constructs such as slider are in many ways just like any other functions in *Mathematica*. They can occur anywhere in an output, in tables, and even inside typeset mathematical expressions. Wherever these functions occur, they carry with them the behavior of dynamically displaying or changing in real time the current value of the expression or variable they are linked to. Dynamic is a simple building block, but the rest of *Mathematica* turns it into a flexible tool for creating nimble, zippy, and often fun little interactive displays.

This makes a table of *x* sliders, which are updated in sync. In[2]:= Table[Slider[Dynamic[x]], {4}] You can combine a slider with a display of its current value in a single output. In[3]:= {Slider[Dynamic[x]], Dynamic[x]} Out[3]= { - , 0. }



Using integer-valued sliders, you can create dynamically updated algebraic expressions.

You can use dynamic expressions with Panel, Row, Column, Grid, and other formatting constructs.



Notice that the last example resembles the output of Manipulate. This is no coincidence, because Manipulate in fact produces a combination of Dynamic, controls, and formatting constructs, not fundamentally different from what you can do yourself using these lower-level functions.

Localizing Variables in Dynamic Output



If you have both these outputs visible and drag either slider, you will notice that they are communicating with each other. Move the slider in one example, and the other example moves too. This is because you are using the global variable *x* in both examples. Although this can be very useful in some situations, most of the time you would probably be happier if these two sliders could be moved independently. The solution is a function called DynamicModule.

DynamicModule [{x,y,},expr]	an object which maintains the same local instance of the symbols <i>x</i> , <i>y</i> , in the course of all evaluations of Dynamic objects in <i>expr</i>
<pre>DynamicModule [{x=x0, y=y0}, expr]</pre>	specifies initial values for x, y,

Localizing and initializing variables for Dynamic objects.

DynamicModule has arguments identical to Module and is similarly used to localize and initialize variables, but there are important differences in how they operate.





Multiple DynamicModules can be placed in a single output, and they maintain separate values of the variables associated with their respective areas in the output.



You might be tempted to use Module in place of DynamicModule, and in fact this would appear to work at first. However, it is not a good idea for several reasons, which are discussed in more detail in "Advanced Dynamic Functionality".

DynamicModule does its work in the front end, not in the kernel. It remains unchanged by evaluation, and when formatted as output, it creates an invisible object embedded in the output expression which handles the localization. As long as that space of output remains in existence (i.e., is not deleted), the invisible object representing the DynamicModule will maintain the values of the variables, allowing them to be used in subsequent evaluations of Dynamic expressions within the scope (area) of the DynamicModule.

If you save a notebook containing a DynamicModule, close that notebook, then later reopen it in a new *Mathematica* session, the values of all the local variables will still be preserved and the sliders inside the DynamicModule will be in the same positions. This will *not* be the case with sliders linked to global variables (like the earliest examples in this tutorial), nor with sliders

Module DynamicModule

linked to variables localized with Module instead of DynamicModule. Such variables store their values in the current *Mathematica* kernel session, and they are lost as soon as you quit *Mathematica*.

In addition to localizing variables to particular regions of output, DynamicModule provides options to automatically initialize function definitions when an expression containing a DynamicModule is opened, and to clean up values when the expression is closed or deleted. More details are found in DynamicModule.

The Second Argument of Dynamic

Dynamic connections are by default bi-directional. Sliders connected to a variable move together because they both reflect and control the value of the same variable. When you drag a slider thumb, the system constructs and evaluates expressions of the form *expr* = *new*, where *expr* is the expression given in the first argument to Dynamic and *new* is the proposed new value determined by where you have dragged the slider thumb. If the assignment can be done, the new value is accepted. If the assignment fails, the slider will not move.

These two sliders move in opposite directions when you move the first one. However, trying to move the second slider gives an error because you cannot assign a new value to the expression 1 - x.

```
In[1]:= DynamicModule[{x = 0}, {Slider[Dynamic[x]], Slider[Dynamic[1 - x]]}]
Out[1]= {-______, _____}
```

You can keep an arbitrary expression in the first argument of Dynamic, but change the dynamically executed evaluation by using the optional second argument. This is a convenient way to specify "inverse functions" that update the values of variables in the first arguments. *Mathematica* does not attempt to deduce such inverse functions automatically from the first argument of Dynamic; you have to supply one yourself.

```
Dynamic [expr, f]continually evaluates f [val, expr] during interactive<br/>changing or editing of valInverse functions.
```

Now the dynamically executed expression in the second slider is the pure function $(x = 1 - \pm) \&$, which is given the proposed new value in \pm . Note that the function is responsible for actually doing the assignment to whatever variable you want to change; you cannot just say $(1 - \pm) \&$ if you want to change x.

The ability to interpose your own arbitrary function between the mouse position and the state of *Mathematica* is very powerful, and you can use it for purposes beyond simple inverse functions. The function given in the second argument is effectively free do to anything it wants.

This defines "detents" that snap the slider to integer values if the thumb is within a certain tolerance of a round number.

This makes the variable take on rational numbers (integer fractions) instead of decimals.

For complete control over the tracking behavior, it is possible to specify separate functions that are called at the start, middle, and end of a mouse click on the slider thumb. If you are familiar with conventional user-interface programming, you will recognize these as separate, high-level event functions for the mouse-down, mouse-drag, and mouse-up events.

The second argument of Dynamic also lets you restrict the movement of a slider and effectively implement geometric constraints.

You can only move the thumb of this Slider2D along a circle.

```
In[34]:= DynamicModule[{pt = {1, 0}},
Slider2D[Dynamic[pt, (pt = # / Norm[#]) &], {-1, 1}, Exclusions → {0, 0}]]
Out[34]=
```

Where Should Dynamic Be Placed in an Expression?

The fundamental behavior of Dynamic is to build a copy of the input expression into the output cell. To be more specific, Dynamic has the attribute HoldFirst and remains unchanged by evaluation.

```
The result of evaluating Dynamic [x + y] is Dynamic [x + y], which you can see by examining
the InputForm representation of the output.
In[35]:= Dynamic[x + y] // InputForm
Out[35]//InputForm= Dynamic[x + y]
```

You do not see Dynamic in ordinary output because, when formatted for display in the front end, Dynamic [x + y] is represented by an object that contains a copy of the unevaluated input (x + y), but displays as the evaluated value of that expression. The Dynamic wrapper is still present in the output, but it is invisible.

Because Dynamic does its work entirely in the front end, you cannot use it inside functions that need to access the value of an expression in order to do their work.





The Plot command needs to have specific numerical values for x to make a plot, but the Dynamic [x] inside the function being plotted does not *evaluate* into anything in the kernel. It remains inert as Dynamic [x], preventing the Plot command from doing anything sensible.

Another way to look at it is that the expression inside a Plot command does not appear directly anywhere in the output. Dynamic is a formatting function that does its work in the front end, not in the kernel, so if it is used in a way where it will never be placed as output, it is probably a mistake.

When combining Dynamic with controls, it is particularly important to get the Dynamic in the right place.

```
This example works as expected; move the slider and the value of x changes.
In[38]:= DynamicModule[{x = 0.5}, {Slider[Dynamic[x]], Dynamic[x]}]
Out[38]= {______, 0.5}
```

```
This example looks good at first, but if you move the slider, x does not change.

In[39]:= DynamicModule[{x = 0.5}, {Dynamic[Slider[x]], Dynamic[x]}]

Out[39]= {______, 0.5}
```

That is because when the Dynamic wrapped around Slider[x] evaluates its contents, the value of x is substituted, and the result is a slider whose first argument is a specific number, with no trace of the variable name left. The slider in this case is a *dynamic* display of a *static* slider.

What is needed is a *static* slider, which contains within it a *dynamic* reference to the value of the variable. In the case of controls, there is a simple rule for where to put the Dynamic. The first argument of any control function, such as Slider, Checkbox, or PopupMenu, will almost always be Dynamic [*var*].

Beyond these cases where Dynamic will *not* work in a particular position, there is often a great deal of flexibility about where to place Dynamic. It is often used as the outermost function in an input expression, but this is by no means necessary, and in more sophisticated applications, Dynamic is usually used deeper in the expression and can even be nested.

This displays a table of ten copies of the value of x.

```
In[40]:= Dynamic[Table[x, {i, 10}]]
Out[40]= {0., 0., 0., 0., 0., 0., 0., 0., 0., 0.}
```

Dynamic is wrapped around the whole expression, so evaluation of the Table command is delayed until the output is displayed in the notebook. Any time the value of x is changed, the Table command will be reevaluated.

The output from this example looks exactly the same.
In[41]:= Table[Dynamic[x], {i, 10}]
Out[41]= {0., 0., 0., 0., 0., 0., 0., 0., 0., 0.}

But in this case the Table command is evaluated immediately, generating a list of ten separate Dynamic expressions, each of which evaluates x separately after the overall result has been placed in the notebook.

When x is changed, the first example sends a single request to the kernel to get the value of Table $[x, \{i, 10\}]$, while the second example sends ten separate requests to the kernel to get the value of x. It might seem that the first example is obviously more efficient, and in this case it is. However, you should also avoid the other extreme, wrapping too many things into a single Dynamic, which can also be inefficient.

This initializes *x* and *y* to set up a new slider connected to the value of *x*.

```
In[6]:= x = 0.5;
In[7]:= y = Plot3D[Sin[nm], {n, 0, 4}, {m, 0, 4}];
```

```
In[8]:= Slider[Dynamic[x]]
```

```
Out[8]=
```

This is a tab view with two groups of dynamic expressions, both showing the dynamic values of x (a simple number) and y (a 3D plot).

```
In[9]:= TabView[{{Dynamic[x], Dynamic[y]}, Dynamic[{x, y}]}]
```



Drag the slider around, and note that the value of x in the first tab updates quite rapidly. On most computers it will be essentially instantaneous. However, updates are more sluggish in the second tab. Each individual Dynamic expression keeps track (quite carefully) of exactly when it might need to be reevaluated in order to remain up-to-date. In the second tab, the output is forcing the whole expression $\{x, y\}$, including the large, slow 3D plot, to be reevaluated every time the value of x changes. By using two separate Dynamic expressions in the first tab, you allow the value of x to be updated without needing to also reevaluate y, which has not actually changed. (You may want to delete the last output before proceeding, as it will slow down any examples containing the global x as long as it is visible on screen.)

It is hard to make blanket statements about where Dynamic should be placed in every case, but generally speaking if you are building a large, complex output where only small parts of it will change, the Dynamic should probably be wrapped just around those parts. On the other hand, if all or most of the output is going to change in response to a single variable changing its value, then it is probably best to wrap Dynamic around the whole thing.

Dynamic in Options

Dynamic can be used on the right-hand side of options, in those cases where the option value will be transmitted to the front end before being used. This is a somewhat subtle distinction related to the discussion in "Where Should Dynamic Be Placed in an Expression".

An option like PlotPoints in plotting commands cannot have Dynamic on the right-hand side, because the plotting command needs to know a specific numerical value before the plot can be generated. Remember that Dynamic has the effect of delaying evaluation until the expression reaches the front end, and in the case of PlotPoints, that is too late since the value is needed right away. On the other hand, options to functions that do their work in the front end can usually, and usefully, accept Dynamic in their option values.

For example, you can control the size of a block of text in two ways.

Dynamic can	be wrapped	around a	whole Style	expression.
-------------	------------	----------	-------------	-------------

```
In[10]:= h = 12;
In[11]:= {Slider[Dynamic[h], {6, 100}], Dynamic[Style["Some Text", FontSize → h]]}
Out[11]= {
```

```
Or Dynamic can be only in the FontSize option value.
In[59]:= {Slider[Dynamic[h], {6, 100}], Style["Some Text", FontSize → Dynamic[h]]}
Out[59]= {
```

There are two potential advantages to putting the Dynamic in the option value. First, suppose the dynamically regenerated expression is very large, for example if the block of text is the entire document, it is inefficient to retransmit it from the kernel to the front end every time the font size is changed, as is necessary if Dynamic encloses the whole expression.

Second, the output of a Dynamic expression is not editable (since it is liable to be regenerated at any moment), which makes the output of the first example non-editable. But the text in the second example can be edited freely since it is ordinary static output: only the option value is dynamic.

Dynamic option values can be also set in the Option Inspector. They are allowed at the cell, notebook, or global level, and in stylesheets. (Note, however, that if you set a dynamic option value in a position where the value will be inherited by many cells, for example in a stylesheet, there can be a significant impact on performance.)

You can set dynamic option values through SetOptions, as well.

```
In[51]:= x = 0;
```

```
In[51]:= SetOptions [EvaluationNotebook[], Background \rightarrow Dynamic[Hue[x]]]
```

Having linked the background color of the notebook to the global variable x, it can now be controlled by a slider or by a program.

```
In[52]:= Slider[Dynamic[x]]
```

Out[52]=

Of course, it is good to be able to return to normal.

In[53]:= SetOptions [EvaluationNotebook[], Background \rightarrow Inherited]

Dynamic and Infinite Loops

If you are not careful, you can easily throw Dynamic into an infinite loop.

```
This counts upwards as fast as possible for as long as it remains on screen.

In[54]:= DynamicModule[{x = 1}, Dynamic[x = x + 1]]
```

This is not a bug (but delete the above output if it is distracting you to have it there).

Because the output is updated and the screen redrawn after each cycle of an infinite loop, it is actually quite a useful thing to be able to do. Generally speaking, the system will remain responsive to typing, evaluation, and so on, even as the infinitely updating Dynamic zips along.

It is also useful to make such a self-triggering Dynamic that stops changing at some point.



If you have a CPU monitor running, you will see that while the slider is dropping there is a small load on the CPU (for redrawing the screen, primarily), but once it reaches zero, the load drops to nothing. The dynamic tracking system has noticed that the value of *x* did not change: therefore, further updating is not necessary until someone changes the value of *x* again (e.g., when you click on the slider). "Advanced Dynamic Functionality" describes in more detail how the dynamic tracking system works.

A Good Trick to Know

Because it has the attribute HoldFirst, Dynamic does not evaluate its first argument. This is fundamental to the workings of Dynamic. but it can lead to a somewhat unexpected behavior.

For example, suppose you have a list of numbers you wish to be able to modify by creating one slider to control each value.

This creates the list and a dynamic display of its current value.

```
In[1]:= data = {.1, .5, .3, .9, .2};
In[2]:= Dynamic[data]
Out[2]= data
```

This attempts to make a table of sliders, one for each element of the list, using list[[i]] to access the individual members.

```
In[3]:= Table[Slider[Dynamic[data[[i]]]], {i, 5}]
```

Surprisingly, this does not work! You can see an error indication around the sliders, they cannot be moved, and the dynamic output above never changes. You might even jump to the conclusion that part extraction syntax cannot be used in this way with controls. Nothing could be further from the truth.

The problem is that the variable *i* was given a temporary value by the Table command, but that value was never used, because Dynamic is HoldFirst.

```
Looking at the InputForm of the table of sliders reveals the problem.

In[59]:= Table[Slider[Dynamic[data[[i]]]], {i, 5}] // InputForm

Out[59]//InputForm= {Slider[Dynamic[data[[i]]]], Slider[Dynamic[data[[i]]]],

Slider[Dynamic[data[[i]]]], Slider[Dynamic[data[[i]]]],

Slider[Dynamic[data[[i]]]]}
```

What is needed is to do a replacement of the variable *i* with its temporary value, even inside held expressions.

This can be done with $/ \hforemath{\, .}$ or with the somewhat peculiar but convenient idiomatic form demonstrated here.

```
In[60]:= Table[With[{i = i}, Slider[Dynamic[data[[i]]]]], {i, 5}]
```

This output shows that Dynamic does in fact work perfectly with part extraction syntax, a very useful property.

Slow Evaluations inside Dynamic

Dynamic wrapped around an expression that will take forever, or even more than just a few seconds, to finish evaluating is a bad thing.

If you evaluate this example, you will have to wait about 5 seconds before seeing the output \$Aborted.

In[61]:= Dynamic[While[True]]

During the wait for the Dynamic output to evaluate, the front end is frozen, and no typing or other action is possible. Because updating of ordinary dynamic output locks up the front end, it is important to restrict the expressions you put inside Dynamic to things that will evaluate relatively quickly (preferably, within a second or so). Fortunately computers, and *Mathematica*, are fast, so a wide range of functions, including complex 2D and 3D plots, can easily be evaluated in a fraction of a second.

To avoid locking up the front end for good, dynamic evaluations are internally wrapped in TimeConstrained, with a timeout value of, by default, 5 seconds. (This can be changed with the DynamicEvaluationTimeout option.) In certain extreme cases, TimeConstrained can fail to abort the calculation, in which case the front end will, a few seconds later, put up a dialog box allowing you to terminate dynamic updating until the offending output has been deleted.

Fortunately there is an alternative if you need to have something slow in a Dynamic. The option SynchronousUpdating → False allows the dynamic to be evaluated in a way that does not lock up the front end. During evaluation of such an asynchronous Dynamic the front end continues operating as usual, but the main Shift+Return evaluation queue is occupied evaluating the Dynamic, so further Shift+Return evaluations will wait until the Dynamic finishes. (Normal synchronous Dynamic evaluations do not interfere with Shift+Return evaluations.)

Evaluate this example, and you will see a gray placeholder rectangle for about 10 seconds, after which the result will be displayed. In[62]:= Dynamic[{DateList[], Pause[10]; DateList[]}, SynchronousUpdating → False]

Out[62]= {{2009, 1, 2, 15, 47, 41.977359}, {2009, 1, 2, 15, 47, 51.981876}}

Importantly, during that 10-second pause you are free to continue working on other things in the front end.

"Advanced Dynamic Functionality" gives more details about the differences between synchronous and asynchronous dynamic evaluations. In general, you should not plan to use asynchronous ones unless is it absolutely necessary. They do not update as quickly, and can interact in a very surprising, though not technically incorrect, way with controls and other synchronous evaluations.

Further Reading

The implementation details behind Dynamic and DynamicModule are worth understanding if you plan to use complex constructions, particularly those involving nested Dynamic expressions. This is discussed in "Advanced Dynamic Functionality".

Advanced Dynamic Functionality

"Introduction to Manipulate" and "Introduction to Dynamic" provide most of the information you need to use *Mathematica*'s interactive features accessible through the functions Manipulate, Dynamic, and DynamicModule. This tutorial gives further details on the workings of Dynamic and DynamicModule and describes advanced features and techniques for achieving maximum performance for complex interactive examples.

Many examples in this tutorial display a single output value and use Pause to simulate slow calculations. In real life, you will instead be doing useful computations and displaying sophisticated graphics or large tables of values.

Please note that this is a hands-on tutorial. You are expected to actually evaluate each of the input lines as you reach them in your reading, and watch what happens. The accompanying text will not make sense without evaluating as you read.

Module versus DynamicModule

Module and DynamicModule have similar syntax and in many respects behave similarly, at least at first glance. They are, however, fundamentally different in such areas as when their variables are localized, where the local values are stored, and in what universe the variables are unique.

Module works by replacing all occurrences of its local variables with new, uniquely named variables, constructed so that they do not conflict with any variables in the current session of the *Mathematica* kernel.

You can see the names of these localized variables by allowing them to "escape" the context of the module without having been assigned a value.

```
In[3]:= Module[{x}, x]
Out[3]= x$651
```

Out[5]= NOUSI

The local variables can be updated dynamically just like any other variables.

```
In[4]:= Module[{x}, Dynamic[x]]
```

Out[4] = x\$653

That is why sliders inside Module seem to work just as well as sliders inside DynamicModule.

Both examples produce seemingly independent sliders that allow separate settings of separate copies of the variable *x*. The problem with sliders inside Module is that a different kernel session may coincidentally share the same localized variable names. So if this notebook is saved and then reopened sometime later, the sliders may "connect" to variables in some other Module that happen to have the same local variables at that time.

This will not happen with the sliders inside DynamicModule because DynamicModule waits to localize the variables until the object is displayed in the front end and generates local names that are unique to the current session of the front end. Localization happens when DynamicModule is first created as output and then repeats anew each time the file that contains DynamicModule is opened, so there can never be a name conflict among examples generated in different sessions.

Variables generated by Module are purely kernel session variables; when the kernel session ends, the values are irretrievably lost. DynamicModule, on the other hand, generates a structure in the output cell that is responsible for maintaining the values of the variables, allowing them to be saved in files. This is a somewhat subtle concept, best explained by way of two analogies. First, you can think of DynamicModule as a sort of persistent version of Module.

Consider this command.

The module in this example evaluates a series of expressions in order, and from one line to the next the values of all the local module variables are preserved (obviously). You can have as many lines as you like in the compound expression, but they all have to be there at the start; once the Module has finished execution, it evaporates along with all its local variables.

DynamicModule, on the other hand, creates an environment in which evaluations of expressions in Dynamic that appear within the body of the DynamicModule are like additional lines in the compound expression in the previous example. From one dynamic update to the next the values of all the variables are preserved, just as if the separate evaluations were separate lines in a compound expression, all within the local variable context created by DynamicModule.

This preservation of variable values extends not just to subsequent dynamic evaluations within the same session, but to all future sessions. Because all the local variable values are stored and preserved in the notebook file, if the notebook is opened in an entirely new session of *Mathematica*, the values will still be there, and dynamic updates will resume just where they left off. DynamicModule is like an indefinitely extendable Module.

Another way to think about the difference between Module and DynamicModule is that while Module localizes its variables for a certain duration of *time* (while the body of the module is being evaluated), DynamicModule localizes its variables for a certain area of *space* in the output.

As long as that space of the output remains in existence, the values of the variables defined for it will be preserved, allowing them to be used in subsequent evaluations of Dynamic expressions within the scope (area) of the DynamicModule. Saving the output into a file puts that bit of real estate into hibernation, waiting for the moment when the file is opened again. (In computer science terms, this is sometimes referred to as a freeze-dried or serialized object.)

The ability of DynamicModule to preserve state across sessions is also a way of extending the notion of editing in a file. Normally when you edit text or expressions in a file, save the file, and reopen it, you expect it to open the way you left it. Editing means changing the contents of a file.

Ordinary kernel variables do not have this property; if you make an assignment to x, then quit and restart *Mathematica*, x does not have that value anymore. There are several reasons for this, not least of which is the question of *where* the value of x should be saved.

DynamicModule answers this question by defining a specific location (the output cell) where values of specific variables (the local variables) should be preserved. Arbitrary editing operations, like moving a slider, typing in an input field, or dragging a dynamic graphics object, change the values of the local variables. And since these values are automatically preserved when the file is saved, the sliders, and other objects, open exactly where they were left. Thus DynamicModule lets you make any quantity editable in the same way that text and expressions can be edited and saved in notebook files.

Front End Ownership of DynamicModule Variable Values

Ordinary variables in *Mathematica* are owned by the kernel. Their values reside in the kernel, and when you ask *Mathematica* to display the value in the front end, a transaction is initiated with the kernel to retrieve the value. The same is true of dynamic output that refers to the values of ordinary variables.



When one slider is moved, the other 499 move in sync with it. This requires 500 separate transactions with the kernel to retrieve the value of x. (The semantics of *Mathematica* are complex enough that there is no guarantee that evaluating x several times in a row will actually return the same value each time: it would not be possible for the front end to improve efficiency by somehow sharing a single value retrieved from the kernel with all the sliders.)

Variables declared with DynamicModule, on the other hand, are owned by the front end. Their values reside in the front end, and when the front end needs a value, it can be retrieved locally with very little overhead.



If a complex function is applied to such a variable, its value must of course be sent to the kernel. This happens transparently, with each side of the system being kept informed on a just-in-time basis of any changes to variable values.

Whether it is better to use a normal kernel variable or a DynamicModule variable in a given situation depends on a number of factors. The most important is the fact that values of all DynamicModule

DynamicModule

DynamicModule

Modulo

DunamiaModula

DynamicModule variables are saved in the file when the notebook is saved. If you need a value preserved between sessions, it must be declared in a DynamicModule. On the other hand, a temporary variable holding a large table of numbers, for example, might be a poor choice for a DynamicModule variable as it could greatly increase the size of the file. It is quite reasonable to nest a Module inside a DynamicModule and vice versa, or to partition variables between the front end and kernel.

In many situations the limiting factor in performance is the time needed to retrieve information from the kernel: by making variables local to the front end, speed can sometimes be increased dramatically.

Automatic Updates of Dynamic Objects

The specification for dynamic output is simple: Dynamic [*expr*] should always display the value you would get if you evaluated *expr* now. If a variable value, or some other state of the system, changes, the dynamic output should be updated immediately. Of course, for efficiency, not every dynamic output should be reevaluated every time any variable changes. It is critical that dependencies be tracked so that dynamic outputs are evaluated only when necessary.

```
Consider these two expressions.

In[9]:= Dynamic[a+b+c]

Out[9]= a+b+c

In[10]:= Dynamic[If[a, b, c]]

Out[10]= If[a, b, c]
```

The first expression might change its value any time the value of a, b, or c changes, or if any patterns associated with a, b, or c are changed. The second expression depends on a and b (but not c) while a is True and on a and c (but not b) while a is False. If a is neither True nor False, then it depends only on a (because the If statement returns unevaluated).

Figuring out these dependencies a priori is impossible (there are theorems to this effect), so instead the system keeps track of which variables or other trackable entities are actually encountered during the process of evaluating a given expression. Data is then associated with those variable(s) identifying which dynamic expressions need to be notified if the given variable receives a new value.

An important design goal of the system is to allow monitoring of variable values by way of dynamic output referencing them, without imposing any more load than absolutely necessary on the system, especially if the value of the variable is being changed rapidly.

Consider this simple example.

```
In[11]:= Dynamic[x]
Out[11]= x
```

```
In[13]:= Do[x, {x, 1, 5000000}]
```

When the dynamic output is created, it is evaluated, and the symbol x is tagged with information identifying the output that needs to be updated if its value should be changed.

When the loop is started and x is first given a new value, the data associated with it is consulted, and the front end is notified that the dynamic output needs to be updated. The data associated with x is then deleted. Essentially the system forgets all about the dynamic output, and subsequent assignments in the loop incur absolutely no speed penalty because of the existence of a dynamic output monitoring the value of x.

Much later (on a computer time scale; only a fraction of a second on a human time scale) when the screen is next redrawn and the dynamic output containing the reference to x is reevaluated, the connection between the dynamic output and the variable x is noticed again, and the association is reestablished.

Meanwhile the loop has continued to run. The next time the assignment is done after the screen is updated, another notification will be sent to the front end, and the process repeats.

By default, dynamic outputs triggered by changes in variable values are updated no faster than twenty times per second (this rate can be changed with the SystemOption "DynamicUpdateInterval"). In the previous example you will typically see the value jump by tens or hundreds of thousands with each update (more the faster your computer is), and the overall speed of the computation is slowed down by only a percent or two, nearly zero if you have a multiprocessor system.

You might expect that having a dynamic output monitoring the value of a symbol that is being changed rapidly in a tight loop would slow that loop down significantly. But the overhead is in fact zero-order in the rate at which the variable is changed, and in practice is usually minimal. Dynamic outputs are only updated when they are visible on screen. This optimization allows you to have an open-ended number of dynamic outputs, all changing constantly, without incurring an open-ended amount of processor load. Outputs that are scrolled off-screen, above or below the current document position, will be left unexamined until the next time they are scrolled on-screen, at which point they are updated before being displayed. (Thus the fact that they stopped updating is not normally apparent, unless they have side effects, which is discouraged in general.)

Dynamic output can depend on things other than variables, and in these cases tracking is also done carefully and selectively.

```
This gives a rapidly updated display of the current mouse position in screen coordinates.
In[14]:= Dynamic[MousePosition[]]
Out[14]= {1058, 553}
```

As long as the output is visible on screen, there will be a certain amount of CPU activity any time the mouse is moved, because this particular dynamic output is being redrawn immediately with every movement of the mouse. But if it is scrolled off-screen, the CPU usage will vanish.

Refresh

Normally, dynamic output is updated whenever the system detects any reason to believe it might need to be (see "Automatic Updates of Dynamic Objects" for details about what this means). Refresh can be used to modify this behavior by specifying explicitly what should or should not trigger updates.

```
This updates when either slider is moved.
```

```
In[15]:= DynamicModule[{x, y}, Column[{
    Slider[Dynamic[x]],
    Slider[Dynamic[y]],
    Dynamic[{x, y}]}]
Out[15]=
(0.245, 0.11)
```

Refresh with a TrackedSymbols option can be used to specify a list of those symbols that should be tracked, with all other reasons for updating being ignored.

This updates only when *x* changes, ignoring changes in *y*.

```
In[16]:= DynamicModule[{x, y}, Column[{
    Slider[Dynamic[x]],
    Slider[Dynamic[y]],
    Dynamic[Refresh[{x, y}, TrackedSymbols → {x}]]}]]
Out[16]=
```

When you move the second (y) slider, nothing happens, but when you move the first slider, the expression is updated to reflect the current value of both variables. You might say that after moving the second slider, the dynamic output is wrong, since it does not reflect the current state of the system. But that is essentially the whole reason for the existence of the Refresh command. It allows you to override the system's mandate to always update dynamic output any time it is potentially out of date.

The setting TrackedSymbols -> Automatic can be used to track only those symbols that occur explicitly (lexically) in the expression given in the first argument to Refresh. For example, if you use a function that depends on a global variable that does not occur lexically inside Refresh, changes to the value of the global variable will not cause updating, when normally they would.

Refresh can also be used to cause updates at regular time intervals. It is important to understand that this is *not* a feature that should be used lightly. It is fundamental to the design of Dynamic that it does not need to update on any fixed schedule, because it simply always updates immediately whenever doing so would be useful. But there are some situations where this either cannot, or just unfortunately does not, happen.

One potentially vexing case is RandomReal. Every time you evaluate RandomReal[], you get a different answer, and you might think that Dynamic[RandomReal[]] should therefore constantly update itself as fast as possible. But this would normally not be useful, and would in fact have negative consequences for a number of algorithms that use randomness internally (e.g., a Monte Carlo integration inside Dynamic should probably not update constantly simply because it will, in fact, give a slightly different answer each time).

For this reason, RandomReal[] is not "ticklish," in the sense that it does not trigger updates. If you want to see new random numbers, you have to use Refresh to specify how frequently you want the output updated. Another example of non-ticklish functions are file system operations.

This gives you a new number every second.

```
In[17]:= Dynamic[Refresh[RandomReal[], UpdateInterval → 1]]
Out[17]= 0.722136
```

This is not updated automatically.

```
In[18]:= Dynamic[FileByteCount[ToFileName[
                 {$TopDirectory, "SystemFiles", "FrontEnd", "Palettes"}, "BasicMathInput.nb"]]]
Out[18]= $Failed
```

In the unlikely event that the file containing the **BasicMathInput** palette changes size, this Dynamic will not be updated. If you want to monitor the size of a file, you need to use Refresh to specify a polling interval. (On sufficiently advanced operating systems it would theoretically be possible for *Mathematica* to efficiently receive notifications of file system activity, and future versions of *Mathematica* might in fact update such expressions automatically. As with other Dynamic expressions, automatic correctness is always the goal.)

Finally, several functions you might think would trigger dynamic updates in fact do not: for example, DateList and AbsoluteTime. As with RandomReal, it would cause more trouble than it is worth for these functions to automatically trigger updates, and Refresh can trivially be used to create clock-like objects. The function Clock is intended specifically as a time-based function that *is* ticklish.

```
This updates approximately every second.
In[19]:= Dynamic[Refresh[DateList[], UpdateInterval → 1]]
Out[19]= {2009, 1, 2, 17, 3, 24.196806}
```

This updates without an explicit Refresh.
In[26]:= Dynamic[Clock[{1, 10}]]

Nesting Refresh

In the Refresh section examples, Refresh is always the outermost function inside Dynamic. You might almost wonder why its options are not simply options to Dynamic. But in fact it is often important to place Refresh as deeply in the expression as possible, especially if it specifies a time-based updating interval.

Consider this example.

When the checkbox is checked, Refresh is causing frequent updating of the clock, and CPU time is being consumed to keep things up-to-date. When the checkbox is unchecked, however, the Refresh expression is no longer reached by evaluation, the output remains static, and no CPU time is consumed. If Refresh were wrapped around the whole expression inside Dynamic, CPU time would be consumed constantly, even if the clock were not being displayed. The words "No clock" would be constantly refreshed, pointlessly. (This refreshing is not visible; there is no flicker of the screen, but CPU time is being consumed nevertheless.)

Nesting Dynamic

Dynamic expressions can be nested, and the system takes great care to update them only when necessary. Particularly when the contents of a Dynamic contain further interactive elements, it is important to keep track of what will stay static and what will update, when a given variable is changed.

```
Consider this example.

In[21]:= DynamicModule[{n = 5, data = Table[RandomReal[], {20}]},

Column[{

Slider[Dynamic[n], {1, 20, 1}],

Dynamic[Column[Table[With[{i = i}, Slider[Dynamic[data[[i]]]]], {i, n}]]]]]

Out[21]=

Out[21]=

Out[21]=
```

The position of the first slider determines the number of sliders underneath it, and each of those sliders in turn is connected to the value of one element of a list of data. Because the number of sliders is variable, and changes dynamically in response to the position of the first slider, the table that generates them needs to be inside Dynamic.

The example works, but now suppose you want to display the value of each number in the list next to its slider.

```
You might at first try this.

In[22]:= DynamicModule[{n = 5, data = Table[RandomReal[], {20}]},

Column[{

Slider[Dynamic[n], {1, 20, 1}],

Dynamic[Grid[Table[With[{i = i},

{Slider[Dynamic[data[[i]]]], data[[i]]}], {i, n}]

]]}]]

Out[22]= 0.0772513

0.0778808

0.627453

0.165515

0.441267
```

Now any time you click one of the lower sliders, it moves only one step, then stops. The problem is that the *data*[[*i*]] expressions in the second column of the grid are creating a dependency in the outer Dynamic on the values in *data*.

As soon as *data* changes, the contents of the outer Dynamic, including the slider you are trying to drag, are destroyed and replaced with a nearly identical copy (in which the displayed value of one of the *data*[[i]] has been changed). In other words, the act of dragging the slider destroys it, preventing any further activity.

The solution to this is to prevent the outer Dynamic from depending on the value of data, by making sure that all occurrences of data in the expression are wrapped in Dynamic. In[23]:= DynamicModule[{n = 5, data = Table[RandomReal[], {20}]},

```
      Im[23]:=
      Dynamicodul [ [ 1 - 3 , duck - Tuble[kandomkedi [ ] , [ 20 ] ] , Column [ 1 Slider[Dynamic [n] , {1, 20, 1}], Dynamic [Grid [Table[With [ {i = i}, { {Slider [Dynamic [data [ [ i ] ] ] ] , Dynamic [data [ [ i ] ] ] }], {i, n}] ] ] ]]]

      Out[23]=
      0.858271

      0.000
      0.858271

      0.000
      0.729104

      0.000
      0.581138

      0.000
      0.779463
```

Now you can drag any of the sliders and see dynamically updated values. This works because the outer Dynamic now depends only on the value of *n*, the number of sliders, not on the value of *data*. (Technically this is because Dynamic is HoldFirst: when it is evaluated, the expression in its first argument is never touched by evaluation, and therefore no dependencies are registered.)

When building large, complex interfaces using multiple levels of nested Dynamic expressions, these are important issues to keep in mind. *Mathematica* works hard to do exactly the right thing even in the most complex cases. For example, the output of Manipulate consists of a highly complex set of interrelated and nested Dynamic expressions: if the dependency tracking system did not work correctly, Manipulate would not work right.

Synchronous versus Asynchronous Dynamic Evaluations

Mathematica consists of two separate processes, the front end and the kernel. These really are separate processes in the computer science sense of the word: two independent threads of execution with separate memory spaces that show up separately in a CPU task monitor.

The front end and kernel communicate with each other through several *MathLink* connections, known as the main link, the preemptive link, and the service link. The main and preemptive links are pathways by which the front end can send evaluation requests to the kernel, and the kernel can respond with results. The service link works in reverse, with the kernel sending requests to the front end.

The main link is used for Shift +Return evaluations. The front end maintains a queue of pending evaluation requests to send down this link. When you use Shift +Return on one or more input cells, they are all added to the queue, and then processed one by one. At any one time, the kernel is only aware of a single main link evaluation, the one it is currently working on (if any). In the meantime, the front end remains fully functional; you can type, open and save files, and so on. There is no arbitrary limit on how long a main link evaluation can reasonably take. People routinely do evaluations that take days to complete.

The preemptive link works the same way as the main link in the sense that the front end can send an evaluation to it and get an answer, but it is administered quite differently on both

Dynamic

ends. On the front end side, the preemptive link is used to handle normal Dynamic updates. There is no queue; instead, the front end sends one evaluation at a time and waits for the result before continuing with its other work. It is thus important to limit preemptive link evaluations to a couple of seconds at most. During any preemptive link evaluation, the front end is completely locked up, and no typing or other actions are possible.

On the kernel side, evaluation requests coming from the preemptive link are given priority over evaluations from the main link, including the current running main link evaluation (if any). If an evaluation request comes from the preemptive link while the kernel is processing a main link evaluation, the main link evaluation is halted at a safe point (usually within microseconds). The preemptive link evaluation is then run to completion, after which the main link evaluation is restarted and allowed to continue as before. The net effect is similar to, though not the same as, a threading mechanism. Multiple fast preemptive link evaluations can be executed during a single long, slow main link evaluation, giving the impression that the kernel is working on more than one problem at a time.

Preemptive link evaluations can change the values of variables, including those being used by a main link evaluation running at the same time. There is no paradox here, and the interleaving is done in a way that is entirely safe, though it can result in some fairly peculiar behavior until you understand what is going on.

For example, evaluate this to get a slider.

In[24]:= Slider[Dynamic[x]] Out[24]=

Then evaluate this command, and during the ten seconds it takes to finish, drag the slider around randomly.

```
In[25]:= Table[Pause[1]; x, {10}]
```

Out[25]= {0, 0.2, 0., 0., 0., 0.5, 0.675, 0., 0., 0.}

You will not see anything happening (other than the slider moving) but when the second evaluation finishes, you will see that it has recorded ten different values of x, representing the positions the slider happened to be at during the ten points at which x was evaluated to build the list.
Dynamic normally uses the preemptive link for its evaluations. Evaluation is synchronous, and the front end locks up until it is finished. This is unavoidable in some cases, but can be suboptimal in others. By setting the option SynchronousUpdating -> False, you can tell the front end to use the main link queue, rather than the preemptive link. The front end then displays a gray box placeholder until it receives the response from the kernel.

In this case, the default (synchronous) update is appropriate because the front end needs to know the result of evaluating the Dynamic [x] for drawing with the correct font size.

```
In[26]:= DynamicModule[{x = 12},
    {Slider[Dynamic[x], {10, 100}], Style["Hello", FontSize → Dynamic[x]]}]
Out[26]= {
```

Here, the output cell is drawn before the second dynamic expression finishes. A gray box placeholder persists for one second until the result is known. Reevaluate the example to see the gray box again.

```
Out[27]= 1
```

Clicking the slider will update the display with a delay of between one and ten seconds. Notice that the cell bracket is outlined, just as if the cell were being Shift +Return evaluated. This is an indication that the evaluation is queued, and that you can continue with other work in the front end while the evaluation is progressing.

Asynchronous updating is useful for displaying full Dynamic subexpressions when it is possible to draw a screen around them and fill in their value later, in much the same way a web browser draws text around an image that is inserted later when it finishes downloading.

Why not always use asynchronous Dynamic expressions? There are several reasons. First, they are queued so that, by definition, they do not operate while another Shift +Return evaluation is underway. This is not the case for normal (synchronous) updates.

A synchronous Dynamic updates smoothly even if the Pause command above is running.
In[28]:= Pause[20]
In[29]:= DynamicModule[{n = 1}, Column[{Slider[Dynamic[n], {1, 10}], Dynamic[n]}]]

```
Out[29]=
```

Also, many controls need to be synchronous in order to be responsive to mouse actions. Making them asynchronous may cause potentially strange interactions with other controls.

```
Here is a problematic example.
In[30]:= n = 1;
Column[{Slider[Dynamic[n], {1, 10}],
        Dynamic[Graphics[Line[Table[{x, Sin[n x]}, {x, 0, 2 Pi, 0.0001}]]],
        SynchronousUpdating → False]}]
Out[31]=
Out[31]=
```

Move the slider around rapidly, and you will end up with a choppy, distorted sine wave, because the value of n changed during the evaluation of the Table command. This is the correct, expected behavior, but it is probably not what you wanted.

This problem does not occur if you use synchronous Dynamic expressions, generally does not happen with DynamicModule local variables, and can be avoided by storing the value of any potentially changing variables into a second variable before starting the asynchronous evaluations.

This fixes the problem.

In[32]:= **n = 1;** Column[{Slider[Dynamic[n], {1, 10}], Dynamic[Module[{n1 = n}, Graphics[Line[Table[{x, Sin[n1 x]}, {x, 0, 2 Pi, 0.0001}]]]], SynchronousUpdating \rightarrow False]}]

ControlActive and SynchronousUpdating→Automatic

As a general rule, if you have a Dynamic that is meant to respond interactively to the movements of a slider or other continuous-action control, it should be able to evaluate in under a second, preferably well under. If the evaluation takes longer than that, you are not going to get satisfactory interactive performance, whether the Dynamic is updating synchronously or asynchronously.

But what if you have an example that simply cannot finish evaluating fast enough, yet you want to be able to make it respond to a slider? One option is to use asynchronous updating and simply accept that you will not get real-time interactive performance. If that is what you want to do, setting ContinuousAction -> False in the slider or other control is a good idea; that way you get only one update after the control is released, avoiding the starting up of potentially lengthy evaluations in the middle of a drag, before you have arrived at the value you want to stop at.

The cell bracket becomes outlined, indicating evaluation activity, only after you release the slider.

```
In[34]:= DynamicModule[{n = 1},
Column[{Slider[Dynamic[n], {1, 10}, ContinuousAction → False],
Dynamic[Pause[n]; n, SynchronousUpdating → False]}]]
Out[34]=
```

Another, much better solution is to provide a fast-to-compute preview of some sort during the interactive control dragging operation, then compute the full, slow output when the control is released. Several features exist specifically to support this.

The first is the function ControlActive, which returns its first argument if a control is currently being dragged, and its second argument if not. Unlike Dynamic, ControlActive is an ordinary function that evaluates in the kernel, returning one or the other of its arguments immediately. It can be embedded inside functions or option values.

The second feature is an option setting SynchronousUpdating -> Automatic for Dynamic, which makes the Dynamic synchronous when a control is being dragged, and asynchronous when the control is released. Together, these two features can be used to implement a fast, synchronously updated display to be used while a control is being dragged, along with a slower, asynchronously updated display when it is released.

The displayed text changes depending on whether or not the slider is being dragged.

```
Out[35]= 
(1, Not Active)
A simple number is displayed, synchronously, while the slider is being dragged, and when it is
released, a graphic is generated asynchronously.
In[36]:= DynamicModule[{n = 3},
Column[{Slider[Dynamic[n], {3, 1000, 1}], Dynamic[Graphics[ControlActive[Inset[n,
{0, 0}], Line[Table[{0, 0}, {Cos[t], Sin[t]}, (t, 0., 2 Pi, 2 Pi / n]]],
TmageSize + 300, PlotRange + 1], SynchronousUpdating → Automatic]}]]
Out[36]=
Out[36]=
Out[36]=
```

This example shows that the front end can remain responsive no matter how long the final display takes to compute and that the preview and the final display can be completely different.

Of course, in most cases, you will want a preview that is some kind of reduced, thinned out, skeletal, or other elided form of the final display. Then the crude form can be fast enough to give a smooth preview, and the computation of the final version, even if it takes awhile, does not block the front end. In fact, this behavior is so useful that it is the default in Plot3D and other plotting functions.

This displays a 3D plot with a very small number of plot points while the control is being dragged and then refines the image with a large number of plot points when the control is released.

```
In[37]:= DynamicModule[{n = 1},
          Column[{Slider[Dynamic[n], {1, 5}], Dynamic[Plot3D[Sin[nxy], {x, 0, 3},
                \{y, 0, 3\}, PlotPoints \rightarrow ControlActive[10, 100], MaxRecursion \rightarrow 0],
              SynchronousUpdating → Automatic]}]]
        1.0
Out[37]= 0.5
         0.0
         -0.5
         -1.0
                                         0
```

3

By default, Plot3D produces a similar preview, though with a somewhat less extreme spread of quality.

```
In[38]:= DynamicModule[{n = 1}, Column[{Slider[Dynamic[n], {1, 5}], Dynamic[
             Plot3D[Sin[n x y], {x, 0, 3}, {y, 0, 3}], SynchronousUpdating \rightarrow Automatic]}]]
```



In addition, Manipulate uses SynchronousUpdating -> Automatic in Dynamic by default so the example becomes as simple as it can be.



In[39]:= Manipulate[Plot3D[Sin[nxy], {x, 0, 3}, {y, 0, 3}], {n, 1, 5}]

You may have noticed one subtlety. When the output of either of the above three examples is first placed in the notebook, you see a crudely drawn (control-active state) version, followed shortly thereafter by a refined (control-inactive) version. This is intentional: the system is providing a fast preview so you see something rather than just a gray rectangle. The first update is done synchronously, just as if a control were being dragged.

This preview-evaluation behavior is examined in more detail in the next section.

ImageSizeCache in Dynamic

ImageSizeCache is an option to Dynamic that specifies a rectangular size to be used in displaying a Dynamic whose value has not yet been computed. It is normally not specified in input, but is instead generated automatically by the front end and saved in files along with the Dynamic expression.

The interaction of ControlActive, SynchronousUpdating, and ImageSizeCache is subtle, complex, and very useful. The first two constructs are explained in ControlActive and SynchronousUpdating→Automatic. The remaining part is explained here.

Note first that Dynamic expressions with the default value of SynchronousUpdating -> True will never have a chance to use the value of their ImageSizeCache option, because they are always computed before being displayed, and, once computed, the actual image size will be used.

On the other hand, Dynamic expressions with SynchronousUpdating \rightarrow False will be displayed as a gray rectangle while they are being computed for the first time. In that case, the size of the rectangle is determined by the value of the ImageSizeCache option. This allows the surrounding contents of the notebook to be drawn in the right place, so that when the Dynamic finishes updating, there is no unnecessary flicker and shifting around of the contents of the notebook. (Users of HTML will recognize this as the analog of the width and height parameters of the img tag.)

It is generally not necessary to specify the ImageSizeCache option explicitly, because the system will set it automatically as soon as the value of the Dynamic is computed successfully. (The computed result is measured, and the actual size copied into the ImageSizeCache option.) This automatically computed value is preserved if the Dynamic output is saved in a file.

```
Consider the following input.

In[40]:= Dynamic[Pause[3]; Style["Hello", 100], SynchronousUpdating \rightarrow False]
Hello", 100], SynchronousUpdating \rightarrow False]
Out[40]= Hello", 100], SynchronousUpdating \rightarrow False]
```

When the input expression is evaluated, a small gray rectangle appears; because this Dynamic has never been evaluated, there is no cache of its proper image size, and a default small size is used.

Three seconds later, the result arrives, and the dynamic output is displayed. At this point an actual size is known, and is copied to the ImageSizeCache option. You can see the value by clicking anywhere in the output cell and choosing **Show Expression** from the **Cell** menu. (This shows you the underlying expression representing the cell, exactly as it would appear in the notebook file if you were to save this cell.) Note the presence of an ImageSizeCache option.

Now type a space in some innocuous place in the raw cell expression (to force a reparsing of the cell contents), and choose **Show Expression** again to reformat the cell. This time you will see a gray rectangle the size of the final output for three seconds, followed by the proper output. This is also what you would see if you opened a notebook containing previously saved, asynchronous dynamic output.

The behavior of the setting SynchronousUpdating -> Automatic is similar, but subtly different. As we saw in the examples in "ControlActive and SynchronousUpdating→Automatic", with the Automatic setting, a synchronous preview-evaluation is done when the output is first placed, to provide a (hopefully) rapid display of the contents of the Dynamic expression before the slower, asynchronous value is computed. Because the first evaluation is synchronous, no gray rectangle is ever displayed.

But this preview evaluation is done only if the ImageSizeCache option is not present. A Dynamic with SynchronousUpdating -> Automatic and an ImageSizeCache option specifying explicit dimensions will not do a synchronous preview evaluation, and will instead display a gray rectangle (of the correct size) pending the result of the first asynchronous evaluation.

This may seem like baffling behavior at first, until you consider the practical effect of it. Generally speaking, Dynamic expressions will always have an ImageSizeCache option (created automatically by the front end) except for the very first time they appear, when they are originally placed as output from an evaluation. Any time they are opened from a file they will have a known, cached size.

In Manipulate, which accounts for the vast majority of dynamic outputs, the default setting is SynchronousUpdating -> Automatic and the described behavior lets the output show up cleanly with a preview image in place when it is first generated. When a file containing dozens of Manipulate outputs is opened, you will get a useful behavior that is familiar from web browsers: the text displays immediately, and graphics (and other dynamic content) fill in later as fast as they are able. So you can scroll through a file rapidly, without any delay associated with precomputing potentially many preview images before the first page of the file can be displayed.

If the initial evaluations when the Manipulate output was first placed were not synchronous, there would be flicker and resizing/shifting of the surroundings, because the size would not be known. But when the Manipulate output is opened from a file, the size is known, and the final output can be placed smoothly without flicker.

One-Sided Updating of ControlActive

After evaluating in the kernel, ControlActive can trigger an update of the Dynamic containing it, but in a highly asymmetric fashion, only when it is going from the active to the inactive state. When making a transition in the other direction, from inactive to active, ControlActive does not trigger any update on its own.

The reason for this somewhat unusual behavior is that ControlActive is a completely global concept. It returns the active state if any control anywhere in *Mathematica* is currently being dragged—even controls that have nothing to do with a particular Dynamic that happen to contain a reference to ControlActive. If ControlActive caused updates on its own, then as soon as you clicked any control, all Dynamic expressions containing references to ControlActive (e.g., a default dynamic Plot3D output) would immediately update, which would be entirely pointless. Instead, only those outputs that have some other reason for updating will pick up the current value of ControlActive.

On the other hand, when the control is released, it is desirable to fix up any outputs that were drawn in control-active form, to give them their final polished appearance. Thus, when ControlActive is going into its inactive state, it needs to, on its own, issue updates to any Dynamic expression that may have been drawn in the active state.

Dragging the slider does not change the Active/Inactive display because ControlActive does not trigger updates on its own.

```
In[49]:= DynamicModule[{x},
    {Slider[Dynamic[x]], Dynamic[ControlActive["Active", "Inactive"]]}]
Out[49]= {
```

This Active/Inactive display updates because *x* in the dynamic output changes.

```
In[41]:= DynamicModule[{x},
        {Slider[Dynamic[x]], Dynamic[{x, ControlActive["Active", "Inactive"]}]}]
Out[41]= {
```

Watch carefully what happens when you click the slider. If you click and hold the mouse without moving it, the display will remain Inactive. But as soon as you move it, the display updates to Active. This is happening because *x* changed, causing the Dynamic as a whole to update, thus picking up the current state of ControlActive.

Now carefully release the mouse button without moving the mouse. Note that the display does revert to Inactive even though x has not changed.

DynamicModule Wormholes

The variables declared in a DynamicModule are localized to a particular rectangular area within one cell in a notebook. There are situations in which it is desirable to extend the scope of such a local variable to other cells or even other windows. For example, you might want to have a button in one cell that opens a dialog box that allows you to modify the value of a variable declared in the same scope as the button that opened the dialog.

This can be done with one of the more surreal constructs in *Mathematica*, a DynamicModule wormhole. DynamicModule accepts the option DynamicModuleParent, whose value is a NotebookInterfaceObject that refers to another DynamicModule anywhere in the front end. For purposes of variable localization, the DynamicModule with this option will be treated as if it resided inside the one referred to, regardless of where the two actually are (even if they are in separate windows).

The tricky part in setting up such a wormhole is getting the NotebookInterfaceObject necessary to refer to the parent DynamicModule. This reference can be created only after the DynamicModule has been created and placed as output, and it is valid only for the current session.

To make the fact avoid all reference process easier, and in to explicit NotebookInterfaceObjectS, DynamicModule also accepts the option InheritScope, which automatically generates the correct value of the DynamicModuleParent option to make the new DynamicModule function as if it were inside the scope of the DynamicModule from which it was created. This is confusing, so an example is in order.

Evaluate this to create an output with a + button and a number.

Clicking the + button increments the value of a DynamicModule local variable, which is displayed at the end of the output. To decrement the number you have to click the **Make** - **Palette** button, which creates a new (very small) floating palette window containing a - button.

This - button is living in a wormhole created by the InheritScope option of the DynamicModule containing it. Clicking the button decrements the value of a local, private variable in the scope of a distant DynamicModule in another window.

InheritScope can be used only when the code creating the second DynamicModule is executed from inside a button or other dynamic object located within the first DynamicModule. By using DynamicModuleParent explicitly, it is possible to link up arbitrary existing DynamicModules, but doing so is tricky, and beyond the scope of this document.

Introduction to Manipulate

The single command Manipulate lets you create an astonishing range of interactive applications with just a few lines of input. Manipulate is designed to be used by anyone who is comfort able using basic commands such as Table and Plot: it does not require learning any complicated new concepts, nor any understanding of user interface programming ideas.

The output you get from evaluating a Manipulate command is an interactive object containing one or more controls (sliders, etc.) that you can use to vary the value of one or more parameters. The output is very much like a small applet or widget: it is not just a static result, it is a running program you can interact with.

This tutorial is designed for people who are familiar with the basics of using the *Mathematica* language, including how to use functions, the various kinds of brackets and braces, and how to make simple plots. Some of the examples will use more advanced functions, but it is not necessary to understand exactly how these work in order to get the point of the example.

Despite the length of this tutorial, it is only half the story. "Advanced Manipulate Functionality" provides further information about some of the more sophisticated features of this rich command.

Manipulate Is as Easy as Table

At its most basic, the syntax of Manipulate is identical to that of the humble function Table. Consider this Table command, which produces a list of numbers from one to twenty.

```
Table[n, {n, 1, 20}]
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}
```

Simply replace the word Table with the word Manipulate, and you get an interactive application that lets you explore values of n with a slider.

Manipulate $[n, \{n, 1, 20\}$]



If you are reading this documentation inside *Mathematica*, you can click and drag the slider to see the displayed value change in real time (meaning that it changes while you are dragging the slider, not just when you release it). If you are reading a static form of the documentation, you will see the slider moved to an arbitrary position. (By default, it starts out on the left side, but in the following examples the slider has typically been moved away from its initial position.)

In both Table and Manipulate, the form {*variable*, *min*, *max*} is used to specify an "iterator", giving the name of the variable and the range over which to vary it.

Of course the whole point of Manipulate (and Table for that matter) is that you can put any expression you like in the first argument, not just a simple variable name. Moving the slider in this very simple output already starts to give an idea of the power of Manipulate.

```
\texttt{Manipulate[Plot[Sin[n x], \{x, 0, 2 Pi\}], \{n, 1, 20\}]}
```



Again, if you are reading this in a static form you will have to trust that the graph changes in real time when the slider is moved.

Note that the slider has an extra icon next to it which, when clicked, opens a small panel of additional controls. Here, the panel from the previous example is opened.



The panel allows you to see the numerical value of the variable, as well as set it in motion using the animation controls.

If you want to see the value of the variable without having to open the subpanel, you can add the option Appearance -> "Labeled" to the variable specification. (Note the number displayed to the right of the plus sign, which is updated in real time as the slider is moved.)





This is also the first hint that Manipulate goes far beyond the relative simplicity of Table, both in its output and in the flexibility and range of what can be specified in the list of variables.

Just like Table, Manipulate allows you to give more than one variable range specification.

```
\begin{array}{l} Manipulate[Plot[Sin[n1 x] + Sin[n2 x], \{x, 0, 2 Pi\}, PlotRange \rightarrow 2], \\ \{n1, 1, 20\}, \{n2, 1, 20\}] \end{array}
```



You can have as many variables as you like, including so many that a similar Table command would try to enumerate an unreasonably large number of entries.





You can open any or all of the subpanels to see numerical values, and you are free to animate many different variables at the same time if you like.

One way to think of Manipulate is as a way to interactively explore a large parameter space. You can move around that space at will, exploring interesting directions as they appear. As you will see in later sections, Manipulate has many features designed to make such exploration easier and more rewarding.

Symbolic Output and Step Sizes

The previous examples are graphical, and indeed the most common application for Manipulate is producing interactive graphics. But Manipulate is capable of making any *Mathematica* function interactive, not just graphical ones.

Often the first issue in examples involving symbolic, rather than graphical, output is that you want to deal with integers, rather than continuously variable real numbers. In Table the default step size is 1, so you naturally get integers, while in Manipulate the default is to allow continuous variation (which you could think of as a step size of zero). Compare these two examples, and note that Manipulate allows values in between those returned by Table.

```
Table[n, {n, 1, 20}]
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}
```

```
Manipulate[n, {n, 1, 20}]
```



Functions involving algebraic manipulations, for example, often do nothing interesting when given noninteger parameter values. This Expand function never expands anything.

```
\texttt{Manipulate}[\texttt{Expand}[(\alpha + \beta)^{n}], \{n, 1, 20\}]
```



Fortunately it is trivial to add an explicit step size of 1 to the Manipulate command, yielding exactly the same set of possible values in Manipulate as is returned by Table.



With an explicit step size, the Expand example is much more interesting.

```
Manipulate[Expand[(\alpha + \beta)^{n}], {n, 1, 20, 1}]
```



The fact that only one value is displayed at a time allows you to create examples that go far beyond what would be practical in a Table command. An important property of Manipulate output is that there is no fixed panel size or arbitrary limit as to how large the output panel can grow.

Manipulate[Expand[$(\alpha + \beta)^{n}$], {n, 1, 300, 1}]

```
 \begin{array}{c} a^{53}+53 \ a^{52} \ \beta+1378 \ a^{51} \ \beta^2+23426 \ a^{50} \ \beta^3+292825 \ a^{49} \ \beta^4+2869 \ 685 \ a^{48} \ \beta^5+22957 \ 480 \ a^{47} \ \beta^6+154 \ 143 \ 080 \ a^{46} \ \beta^7+886 \ 322710 \ a^{45} \ \beta^8+4431 \ 613 \ 550 \ a^{44} \ \beta^9+19 \ 499 \ 099 \ 620 \ a^{43} \ \beta^{10}+76223 \ 753 \ 060 \ a^{42} \ \beta^{11}+266 \ 783 \ 135710 \ a^{41} \ \beta^{12}+841 \ 3929 \ 66470 \ a^{40} \ \beta^{13}+2403 \ 979 \ 904 \ 200 \ a^{39} \ \beta^{14}+6250 \ 3477 \ 750 \ 920 \ a^{38} \ \beta^{15}+119 \ 032 \ 3579 \ 903 \ 435 \ a^{37} \ \beta^{16}+32 \ 308 \ 782 \ 859 \ 535 \ a^{36} \ \beta^{17}+641 \ 828 \ 055 \ a^{32} \ \beta^{21}+462 \ 525 \ 733 \ 568 \ 080 \ a^{31} \ \beta^{22}+623 \ 404 \ 249 \ 591 \ 760 \ a^{30} \ \beta^{23}+779 \ 255 \ 311 \ 989 \ 700 \ a^{29} \ \beta^{24}+903 \ 936 \ 161 \ 908 \ 052 \ a^{28} \ \beta^{25}+973 \ 469 \ 712 \ 824 \ 056 \ a^{27} \ \beta^{26}+973 \ 462 \ 525 \ 733 \ 568 \ 080 \ a^{31} \ \beta^{22}+623 \ 404 \ 249 \ 591 \ 760 \ a^{30} \ \beta^{23}+779 \ 255 \ 311 \ 989 \ 700 \ a^{24} \ \beta^{29}+623 \ 462 \ 525 \ 733 \ 568 \ 080 \ a^{22} \ \beta^{31}+317 \ 986 \ 441 \ 828 \ 055 \ a^{21} \ \beta^{32}+223 \ 423 \ 563 \ a^{30} \ a^{35}+323 \ 323 \ 878 \ 285 \ 535 \ a^{17} \ \beta^{36}+14 \ 844 \ 575 \ 908 \ 435 \ a^{26} \ \beta^{37}+6250 \ 347 \ 750 \ 920 \ a^{18} \ \beta^{35}+323 \ 2308 \ 782 \ 855 \ 53^{11} \ 987 \ 700 \ a^{24} \ \beta^{39}+841 \ 392 \ 966 \ 470 \ a^{13} \ \beta^{34}+64 \ 617 \ 565 \ 719 \ 970 \ a^{18} \ \beta^{35}+323 \ 2308 \ 782 \ 855 \ 535 \ a^{17} \ \beta^{36}+14 \ 844 \ 575 \ 908 \ 435 \ a^{16} \ \beta^{37}+6250 \ 347 \ 750 \ 920 \ a^{15} \ \beta^{38}+223 \ 2308 \ 782 \ 855 \ 535 \ a^{17} \ \beta^{36}+14 \ 844 \ 575 \ 908 \ 435 \ a^{16} \ \beta^{37}+6250 \ 347 \ 750 \ 920 \ a^{15} \ \beta^{38}+23 \ 2308 \ 782 \ 855 \ 535 \ a^{17} \ \beta^{36}+14 \ 844 \ 575 \ 908 \ 435 \ a^{16} \ \beta^{37}+6250 \ 347 \ 750 \ 920 \ a^{15} \ \beta^{38}+223 \ 230 \ 876 \ 855 \ a^{17} \ \beta^{36}+14 \ 844 \ 575 \ 908 \ 435 \ a^{16} \ \beta^{37}+250 \ 347 \ 750 \ 920 \ a^{15} \ \beta^{38}+220 \ 357 \ a^{16} \ \beta^{37}+23 \ a^{16} \ \beta^{37}+153 \ a^{16} \ \beta^{37}+153 \ a^{16} \ \beta^{37}+153 \ a^{16} \ \beta^{37}+153 \ a^{16} \ \beta^{37}+153
```

(In printed forms of this documentation, the slider is set fairly low to avoid wasting paper, but when moved all the way to the right, the output smoothly grows to cover many pages worth of vertical space.)

As with Table, if you use rational numbers for the minimum and step, you will get perfect rational numbers in the variable, not approximate real numbers. Here is an example that uses the formatting function Row to create a simple example of adding fractions.



You can even use end points and step sizes that are symbolic expressions rather than just plain numbers.

 $\label{eq:manipulate} \verb"Manipulate[Row[{n, "+", m, "=", n+m}], {n, a, 10 a, a / 12}, {m, a, 10 a, a / 12}]$



Types of Controls

Manipulate supports a wide range of alternate ways of specifying variables, which generate different kinds of controls for those variables. This includes checkboxes, popup menus, and others in addition to sliders.

The principle is that for each variable, you ask for a particular set of possible values, and Manipulate automatically chooses an appropriate type of control to make those values conveniently available. For a typical numerical Table-like iterator, a slider is the most convenient interface.

You might, on the other hand, want to specify a discrete list of possible values (numeric or symbolic) rather than a range. This is done with an iterator of the form {*variable*, {*val1*, *val2*, ...}}.

(Note the extra level of list compared to the range specification.) If you ask for a small number of separate values, you will get a row of buttons.





If you ask for a larger number of discrete values, Manipulate will switch to using a popup menu.

```
\begin{array}{l} Manipulate[Plot[Sin[n1 x] + Sin[n2 x], {x, 0, 2 Pi}, Filling \rightarrow filling, PlotRange \rightarrow 2], \\ {n1, 1, 20}, {n2, 1, 20}, \\ {filling, {None, Axis, Top, Bottom, Automatic, 1, 0.5, 0, -0.5, -1}}] \end{array}
```



If you use the specific values True and False, you will get a checkbox.

```
\begin{array}{l} Manipulate[Plot[Sin[n1 x] + Sin[n2 x], \{x, 0, 2 Pi\}, Frame \rightarrow frame, PlotRange \rightarrow 2], \\ \{n1, 1, 20\}, \{n2, 1, 20\}, \{frame, \{True, False\}\} \end{array}
```



These choices are of course somewhat arbitrary, but they are designed to be convenient, and you can always override the automatic choice of control type using a ControlType option inserted into the variable specification. (The full list of possible control types is given in the documentation for Manipulate.)

For example, you can ask for a row of buttons even if the automatic behavior would have chosen a popup menu, using the option ControlType -> SetterBar.

 $\begin{array}{l} Manipulate[Plot[Sin[n1 x] + Sin[n2 x], \{x, 0, 2 Pi\}, Filling \rightarrow filling, PlotRange \rightarrow 2], \\ \{n1, 1, 20\}, \{n2, 1, 20\}, \{filling, \\ \{None, Axis, Top, Bottom, Automatic, 2, 1, 0, -1, -2\}, ControlType \rightarrow SetterBar\}] \end{array}$



54 | Dynamic Interactivity

Sliders can be used to scan through discrete symbolic values, not just through numerical ranges (and this allows you to animate through them as well). The option ControlType -> Manipulator asks for the default control used by Manipulate, which is a slider plus an optional control panel with numerical value and animation controls (see the previous example). ControlType -> Slider asks for a plain slider.

 $\begin{array}{l} Manipulate[Plot[Sin[n1 x] + Sin[n2 x], \{x, 0, 2 Pi\}, Filling \rightarrow filling, PlotRange \rightarrow 2], \\ \{n1, 1, 20\}, \{n2, 1, 20\}, \{filling, \{None, Axis, Top, Bottom, \\ Automatic, 1, 0.5, 0, -0.5, -1\}, ControlType \rightarrow Manipulator\}]\end{array}$



It is even possible to use two different controls to adjust the value of the same variable. Here both a popup menu and a slider are connected to the value of the filling variable. If the slider is used to select a value that does not appear in the popup menu, the popup will appear blank, but remains functional. When a value is chosen from the popup menu, the slider is moved to the corresponding position. Both controls can thus be used interchangeably to adjust the same value, and each one follows along when the other is being used.

 $\begin{array}{l} Manipulate[Plot[Sin[n1 x] + Sin[n2 x], {x, 0, 2 Pi}, Filling \rightarrow filling, PlotRange \rightarrow 2], \\ {n1, 1, 20}, {n2, 1, 20}, \\ {filling, {None, 2, 1.5, 1, 0.5, 0, -0.5, -1, -1.5, -2}}, {filling, -2, 2}] \end{array}$



This is not an exhaustive list of the possible control types in Manipulate. See the Manipulate documentation for a more detailed listing. One of the most important control types, Locator, which allows you to place control points inside graphical output in a Manipulate, is discussed in "Locator", Slider2D is discussed in the "2D Sliders" section.

Initial Values and Labels

Here is a fun example for making Lissajous figures.



Unfortunately you see nothing at first: until you move the a1 and a2 (amplitude) variables away from their initial values of zero, there is nothing to see. It would be convenient to set their initial value to something other than the default left-most value. This is done by using a variable specification of the form { {var, init }, min, max }.

Here is the same example with both amplitudes set to 1 initially, and the default frequency values set to give a pleasing initial figure.

```
\texttt{Manipulate[ParametricPlot[{al Sin[n1 (x + p1)], a2 Cos[n2 (x + p2)]}},
   \{x, 0, 20 \text{ Pi}\}, \text{ PlotRange} \rightarrow 1, \text{ PerformanceGoal} \rightarrow \text{"Quality"}\}, \\ \{n1, 1, 4\}, \{\{a1, 1\}, 0, 1\}, \{p1, 0, 2 \text{ Pi}\}, \\ \{\{n2, 5/4\}, 1, 4\}, \{\{a2, 1\}, 0, 1\}, \{p2, 0, 2 \text{ Pi}\}\} 
                                                                                              0
                                                               -
   n1 =
   a1
                                                               - 0
                                                                 - 8
   p1
   n2
                                                                 - 8
                                                               1 0
   a2
   p2
                                                                 - 8
                                              1.0
                                             0.5
                            0.5
                                                                     0.5
      -1.
                                                                                           .0
                                             -0.5
```

It is fun to watch how one shape turns into another, and in this connection it is good to know about an unusual feature of sliders in *Mathematica*. If you hold down the Option key (Macintosh) or Alt key (Windows), the action of the slider will be slowed down by a factor of 20 relative to the movements of the mouse. In other words, when you drag the mouse left and right, the thumb will move only 1/20th as much as it normally would. If you move outside the area of the slider, the value will start moving slowly in that direction as long as the mouse remains clicked.

By holding down the Shift or Ctrl keys, or both, in addition to the Option/Alt key, you can slow the movement down by additional factors of 20 (one for each additional modifier key). With all three held down, it is possible to move the thumb by less that one part per million of its full range, which can be helpful in examples like this where beautiful patterns are hidden in very small ranges of parameter space.

(The option PerformanceGoal -> "Quality" is used in this example to ensure that ParametricPlot draws smooth curves even when a slider is being moved: the need for this option is explained in more detail in "Advanced Manipulate Functionality".)

By default Manipulate uses the names of the variables to label each control. But you may want to provide longer, more descriptive labels, which can be done by using variable specifications of the form {{var, init, label}, min, max}.

Here is the same example with labels.

```
Manipulate[ParametricPlot[{al Sin[n1 (x + p1)], a2 Cos[n2 (x + p2)]},
    {x, 0, 20 Pi}, PlotRange \rightarrow 1, PerformanceGoal \rightarrow "Quality"],
  {{n1, 1, "Frequency 1"}, 1, 4}, {{a1, 1, "Amplitude 1"}, 0, 1},
  {{p1, 0, "Phase 1"}, 0, 2 Pi}, {{n2, 5/4, "Frequency 2"}, 1, 4},
{{a2, 1, "Amplitude 2"}, 0, 1}, {{p2, 0, "Phase 2"}, 0, 2 Pi}]
                                                          O
  Frequency 1
                                                - 13
  Amplitude 1 =
                                                 13
     Phase 1
  Frequency 2
  Amplitude 2
     Phase 2
                            0.5
                 -0.5
                                          0.5
    -1
                           -0.5
```

Beautifying the Control Area

Manipulate supports a number of features that allow you to rearrange, annotate, and generally pretty up the control area, to make it suit the needs of a particular example. (Advanced users should remember, however, that Manipulate is by no means the only way to create interactive interfaces in *Mathematica*, and if you cannot do what you want using Manipulate, you can easily start using functions such as Dynamic and DynamicModule directly to create free-form, open-ended user interfaces not tied to the particular conventions of Manipulate. These features are explained in detail in "Introduction to Dynamic" and "Advanced Dynamic Functional-ity".)

When you have a small number of controls, it is usually most convenient to have them above the content area of the Manipulate panel. But because screens are typically wider than they are tall, if you have a large number of controls, you may find it better to put them on the left side, using the ControlPlacement option.

```
Manipulate[ParametricPlot[{al Sin[n1 (x + p1)], a2 Cos[n2 (x + p2)]}, {x, 0, 20 Pi},
PlotRange → 1, PerformanceGoal → "Quality"], {{n1, 1, "Frequency 1"}, 1, 4},
{{a1, 1, "Amplitude 1"}, 0, 1}, {{p1, 0, "Phase 1"}, 0, 2 Pi},
{{n2, 5 / 4, "Frequency 2"}, 1, 4}, {{a2, 1, "Amplitude 2"}, 0, 1},
{{p2, 0, "Phase 2"}, 0, 2 Pi}, ControlPlacement → Left]
```



When ControlPlacement is used at the level of the Manipulate as a whole, it sets the default position of all the controls. But the option can also be used inside individual variable specifications, allowing you to distribute controls to multiple sides of the output field.

In the following example the controls naturally fall into two groups of three, or three groups of two. You can use the keyword Delimiter inserted in the sequence of variable specifications to indicate where you would like dividing lines put. Here two unlabeled delimiters break the controls up into three groups.

```
 \begin{array}{l} Manipulate[ParametricPlot[{al Sin[n1 (x + p1)], a2 Cos[n2 (x + p2)]}, \\ & \{x, 0, 20 Pi\}, PlotRange \rightarrow 1, PerformanceGoal \rightarrow "Quality"], \\ & \{\{n1, 1, "Frequency 1"\}, 1, 4\}, \{\{n2, 5 / 4, "Frequency 2"\}, 1, 4\}, \\ & Delimiter, \{\{a1, 1, "Amplitude 1"\}, 0, 1\}, \{\{a2, 1, "Amplitude 2"\}, 0, 1\}, \\ & Delimiter, \{\{p1, 0, "Phase 1"\}, 0, 2 Pi\}, \\ & \{\{p2, 0, "Phase 2"\}, 0, 2 Pi\}, ControlPlacement \rightarrow Left] \end{array}
```



Alternately strings, or delimiters and strings, can be used to label the groups of controls.

```
Manipulate[ParametricPlot[{al Sin[n1 (x + p1)], a2 Cos[n2 (x + p2)]},
     {x, 0, 20 Pi}, PlotRange \rightarrow 1, PerformanceGoal \rightarrow "Quality"],
  {A, 0, 2011}, floting → i fortenancecerit, a},
"Horizontal", {{n1, 1, "Frequency"}, 1, 4},
{{a1, 1, "Amplitude"}, 0, 1}, {{p1, 0, "Phase"}, 0, 2 Pi},
Delimiter, "Vertical", {{n2, 5 / 4, "Frequency"}, 1, 4},
{{a2, 1, "Amplitude"}, 0, 1}, {{p2, 0, "Phase"}, 0, 2 Pi}, ControlPlacement → Left]
 Horizontal
 Frequency
                                                               - 8
 Amplitude
                                                               ի 🛛
      Phase
                                                                                                          0.5
 Vertical
                                                                 Frequency =
                                                                                           -0.5
                                                                                                                               0.5
                                                                                                                                                  .0
                                                                         -1.
 Amplitude
      Phase
                                                                 -0.5
```

Quite a variety of things can be interspersed with the controls, including styled text, arbitrary expressions, and even dynamic objects that update independently of the main output window. Here is a simple example of using style to make the group headings more prominent.

```
Manipulate[ParametricPlot[{al Sin[n1 (x + p1)], a2 Cos[n2 (x + p2)]},
   {x, 0, 20 Pi}, PlotRange \rightarrow 1, PerformanceGoal \rightarrow "Quality"],
 Style["Horizontal", 12, Bold], {{n1, 1, "Frequency"}, 1, 4},
 {{a1, 1, "Amplitude"}, 0, 1}, {{p1, 0, "Phase"}, 0, 2Pi},
Delimiter, Style["Vertical", 12, Bold], {{n2, 5/4, "Frequency"}, 1, 4},
 \{\{a2, 1, "Amplitude"\}, 0, 1\}, \{\{p2, 0, "Phase"\}, 0, 2Pi\}, ControlPlacement \rightarrow Left\}
 Horizontal
 Frequency
 Amplitude
    Phase
                                                                       0.5
 Vertical
 Frequency =
                                                            -0.5
                                                                                    0.5
                                                                                                 .0
                                          6
 Amplitude
    Phase
                                                                       -0.5
```

Examples of more complex arrangements and dynamic labels are shown in "Advanced Manipulate Functionality".

2D Sliders

A clever feature of *Mathematica* is support for two-dimensional sliders, which allow you to use both directions of mouse movement to control two values simultaneously. (Ordinary onedimensional sliders in a sense waste one of the two degrees of freedom a mouse is capable of.)

To get a 2D slider, use pairs of numbers for both the *min* and *max*, as in $\{var, \{x_{min}, y_{min}\}, \{x_{max}, y_{max}\}\}$

The value of the variable will also be an $\{x, y\}$ pair. In this trivial example, just look at the value of the variable to get a feel for how the control works.

```
Manipulate[pt, {pt, {-1, -1}, {1, 1}}]
```



The following example shows more graphically how the value of a 2D slider corresponds to a coordinate point.

```
\label{eq:manipulate_graphics} \begin{split} & \texttt{Manipulate[Graphics[{PointSize[0.1], Point[pt]}, PlotRange \rightarrow 1],} \\ & \texttt{[pt, \{-1, -1\}, \{1, 1\}]} \end{split}
```



To do something more interesting, you can recast the Lissajous figure from the previous section with three 2D sliders instead of six 1D sliders. You are controlling the same six parameters, but now you can do it two at a time.

```
Manipulate[
ParametricPlot[{a[[1]] Sin[n[[1]] (x + p[[1]])], a[[2]] Cos[n[[2]] (x + p[[2]])]},
    {x, 0, 20 Pi}, PlotRange → 1, PerformanceGoal → "Quality"],
    {{n, {1, 5 / 4}, "Frequency"}, {1, 1}, {4, 4}},
    {{a, {1, 1}, "Amplitude"}, {0, 0}, {1, 1}},
    {{p, {0, 0}, "Phase"}, {0, 0}, {2 Pi, 2 Pi}}, ControlPlacement → Left]
```



This creates an example that is compact and fun. Note that fine control using the Option, Shift, and Ctrl keys to slow down the motion of sliders (as explained in "Initial Values and Labels") works for 2D sliders as well as 1D sliders.

Graphics beyond Plotting

So far high-level plotting functions have mostly been used, but it is equally interesting to use *Mathematica*'s low level graphics language inside Manipulate. The following example, repeated from the previous section, is a trivial example of using the low-level graphics language.

```
 \begin{array}{l} \texttt{Manipulate[Graphics[{PointSize[0.1], Point[pt]}, PlotRange \rightarrow 1], \\ \texttt{[pt, \{-1, -1\}, \{1, 1\}\}]} \end{array} \end{array}
```



This example also makes the important point that anytime you use Graphics inside Manipulate, you probably want to set an explicit PlotRange option. (PlotRange -> 1 means 1 in all directions from the origin, and is equivalent to PlotRange -> {{-1, 1}, {-1, 1}}.) If you omit the PlotRange option *Mathematica*'s automatic plot range determination will cause the dot to appear not to move at all, because the plot range is always exactly centered around it.

Simple (or complicated) *Mathematica* programming can add arbitrary graphical elements to the output. For example, here we have lines to the center point instead of a dot, with a second linear slider determining the number of lines.

```
\begin{array}{l} Manipulate[ \\ Graphics[{Line[Table[{{Cos[t], Sin[t]}, pt}, {t, 2. Pi / n, 2. Pi, 2. Pi / n}]]}, \\ PlotRange \rightarrow 1], {{n, 30}, 1, 200, 1}, {pt, {-1, -1}, {1, 1}}] \end{array}
```



Here is a fun little string-figure example also based on creating a table of lines.

Because *Mathematica* is a sophisticated programming language, it is possible to use Manipulate to explore parameterized programs or algorithms interactively. The *Mathematica* graphics language is explained in "The Structure of Graphics", and many more examples like this can be found in The Wolfram Demonstrations Project.

Locator

For creating interactive graphics examples, one of the most important features of Manipulate is the ability to place a control point, called a Locator, inside graphics that appear in the output area.
Consider the previous example with lines going to a center point. While using a 2D slider is a fine way to control the center point, you might prefer to be able to simply click and drag the center point itself. This can be done by adding Locator to the control specification for the *pt* variable. In this case it is not necessary to specify a *min* and *max* range, because it can be taken automatically from the graphic. (It is, however, necessary to specify an initial value.)

```
Manipulate[
    Graphics[{Line[Table[{{Cos[t], Sin[t]}, pt}, {t, 2. Pi / n, 2. Pi, 2. Pi / n}]]},
    PlotRange → 1], {{n, 30}, 1, 200, 1}, {{pt, {0, 0}}, Locator}]
```



Now you can click anywhere in the graphic and the center point of the lines will follow the mouse as long as you keep the mouse button down. (It is not necessary to click exactly on the center; it will jump to wherever you click, anywhere in the graphic.)

You can have multiple Locator controls by listing them individually, and it is perfectly fine to have a Manipulate with no controls outside the content area, so you can create purely graphical examples.

```
\begin{array}{l} Manipulate[Graphics[Polygon[{pt1, pt2, pt3}], PlotRange \rightarrow 1], \\ & \{ \{pt1, \{0, 0\}\}, Locator\}, \{ \{pt2, \{0, 1\}\}, Locator\}, \{ \{pt3, \{1, 0\}\}, Locator\} \} \end{array}
```



When there are multiple locators, you can still click anywhere in the graphic, and the nearest Locator will jump to where you click and start tracking the mouse.

Instead of using multiple separate variables, each of which corresponds to a single $\{x, y\}$ point, you can use a single variable whose value is a list of points.



Manipulate[Graphics[Polygon[pts], PlotRange \rightarrow 1], {{pts, {{0, 0}, {1, 0}, {0, 1}}}, Locator}]

Again, if you click anywhere in the graphic, not on a particular Locator, the nearest one will jump to the mouse and start tracking it.

Due to internal limitations, it is not possible to combine individual Locator variables with a variable that is a list of multiple Locator variables: you can have only one multipoint Locator variable in a Manipulate. However, in exchange, it is possible to add the option LocatorAutoCreate -> True to that one Locator multivariable specification, and thereby allow you to create and destroy Locator points interactively (changing the length of the list of points stored in the variable).

In the following example, hold down the Cmd key (Macintosh) or Alt key (Windows) and click anywhere that is not an existing Locator to create a new one at that location. Cmd/Alt click an existing Locator to destroy it. When you add or remove a Locator, you are changing the length of the list of points stored in the *pts* variable, thus changing the number of vertices in the displayed polygon.

```
Manipulate[Graphics[Polygon[pts], PlotRange → 1],
        {{pts, {{0, 0}, {.5, 0}, {0, .5}}}, Locator, LocatorAutoCreate → True}]
```



You can of course combine Locator controls with normal Manipulate variables. For example, you can use some sliders and color choosers to control the appearance of the polygon.

```
Manipulate[
Graphics[{FaceForm[face], EdgeForm[{edge, Thickness[thickness]}], Polygon[pts]},
PlotRange → 1, Background → background],
{face, Green},
{edge, Red},
{background, Cyan},
{(thickness, 0.02}, 0, 0.1},
{{pts, {{0, 0}, {.5, 0}, {0, .5}}}, Locator, LocatorAutoCreate → True}]
```



While a case can be made that the previous examples are frivolous, they are meant to demonstrate the generality of the system: it provides a framework inside of which anything is possible. And the following example shows that even just a couple of lines of code can do something quite remarkable: create an interactive polynomial curve-fitting environment.

The locator thumbs represent data points that are being fit by least squares with a polynomial whose order is determined by the "order" slider. Five points are provided initially, but you can add new ones by Cmd/Alt clicking any blank area of the graphic, or remove one by Cmd/Alt clicking it.

```
Manipulate[Module[{x}, Plot[Fit[points, Table[x^i, {i, 0, order}], x],
        {x, -2, 2}, PlotRange → 2, ImageSize → 500, Evaluated -> True]],
        {{order, 3}, 1, 10, 1, Appearance → "Labeled"},
        {{points, RandomReal[{-2, 2}, {5, 2}]}, Locator, LocatorAutoCreate → True}]
```



The fact that an example of this sophistication can be constructed using such a small volume of code is really quite remarkable. And if you want to really impress someone with the compactness of *Mathematica* code, the following example shows how to do it using only two lines, with some loss of generality. Practice a bit and you can type this from scratch in 30 seconds or less.

```
\begin{aligned} &\texttt{Manipulate[Plot[InterpolatingPolynomial[points, x], {x, -2, 2}, PlotRange \rightarrow 2], \\ & \{\{\texttt{points, RandomReal[{-2, 2}, {5, 2}]\}, Locator\}] \end{aligned}
```



3D Graphics

Manipulate can be used to explore 3D graphics just as easily as 2D, though performance issues become more of a concern. Consider this simple example.



Manipulate[Plot3D[Sin[nxy], {x, 0, 3}, {y, 0, 3}], {n, 1, 5}]

For large values of n the function oscillates rapidly, and in order to produce a smooth picture, the default adaptive sampling algorithm in Plot3D produces a fairly large number of polygons, with correspondingly long computation and rendering times.

Fortunately, Plot3D and other built-in plotting functions automatically adjust their internal algorithms and settings when used inside Manipulate in order to deliver increased speed while a control is being dragged, sometimes at the expense of rendering quality. As soon as the mouse button is released, a high-quality version of the plot is generated asynchronously (meaning other operations in the front end can continue while the plot is being generated). Asynchronous evaluations are discussed in further detail in "Synchronous Versus Asynchronous Dynamic Evaluations" in "Advanced Dynamic Functionality".

The net result is that while you drag the slider, a fast, but somewhat crude, rendering of the plot is created in real time, and when you release the control, a smooth rendering shows up a moment later. (This happens because Plot3D, and most other plotting functions, refer to the function ControlActive in the default settings of the various options that control rendering quality and speed. See "Dealing with Slow Evaluations" in "Advanced Manipulate Functionality" for more about using ControlActive within Manipulate.)

As in 2D, you can use the low-level graphics language just as easily as higher-level plotting commands. In this example you can see how *Mathematica* handles spheres that intersect with each other and with the bounding box.





This example shows how opacity (which is to say, transparency) can be used to see inside nested 3D structures.

```
Manipulate[SphericalPlot3D[\theta + \phi, {\theta, 0, a \pi}, {\phi, 0, b \pi}, SphericalRegion \rightarrow True, PlotRange \rightarrow 10, Ticks \rightarrow None, BaseStyle \rightarrow Opacity[opacity]], {a, 0.1, 2}, {b, 0.1, 2}, {opacity, 1, 0}]
```



(Note that adding transparency to a 3D graphic can slow down rendering significantly.)

You can rotate a 3D graphic inside a Manipulate output by clicking and dragging it in the ordinary way. In most cases if you subsequently move one of the Manipulate controls, the graphic will stay rotated to the position you moved it to manually, unless the graphics expression in the Manipulate contains an explicit ViewPoint option, or wraps the graphical output in additional formatting constructs.

All Types of Output Are Supported

Manipulate is designed to work with the full range of possible types of output you can get with *Mathematica*, and it does not stop with graphical and algebraic output. Any kind of output supported by *Mathematica* can be used inside Manipulate. Here are some examples which may be less than obvious.

Formatting constructs such as Grid, Column, Panel, etc. can be used to produce nicely formatted outputs. (See "Grids, Rows, and Columns" for more information about formatting constructs.)



 $\begin{array}{l} Manipulate[Grid[Table[{i, i^m}, {i, 1, n}], Alignment \rightarrow Left, Frame \rightarrow All], \\ {n, 1, 20, 1}, {m, 1, 100, 1}] \end{array}$

You can even wrap Manipulate around functions that generate user interface elements like sliders and tab views. (See "Control Objects" and "Viewers and Annotation" for more information about user interface elements.) In this example we use two sliders to control the appearance of a third slider.

```
Manipulate[Column[{{style, size}, Slider[0.5, Appearance → {style, size}]}],
{style, {"Automatic", "Vertical", "LeftArrow",
    "RightArrow", "UpArrow", "DownArrow"}, ControlType → Slider},
{size, {"Automatic", "Tiny", "Small", "Medium", "Large"}, ControlType → Slider}]
```

In this more complicated example the structure of a TabView is controlled by a Manipulate. Dynamic [pane] allows the current pane of the TabView to be selected either by using the slider created by Manipulate, or by clicking the TabView in the output area. The output is fully active.



This example may be somewhat alarming, but is meant only to illustrate that Manipulate is a fully general function, not limited to exploring any fixed domain of graphical or algebraic examples. There is literally nothing you can see in a cell in a *Mathematica* notebook that you cannot interactively explore using Manipulate (subject only, of course, to the speed of your computer).